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The correlation of microtremors: empirical limits and relations between results in frequency and time domains

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SUMMARY

The use of the correlation of microtremor records is on its way to develop into a common tool to estimate local shear wave velocity structure. For this reason, the establishment of the conditions for the correct use of this method and its limitations when applied to real data is becoming increasingly important. In addition to the use of frequency domain spatial correlation technique [the Spatial Autocorrelation (SPAC) method], the use of time domain correlation to obtain the Green's function of the medium is rapidly gaining presence. We explore the use of microtremor correlation techniques in the time domain to determine local velocity structure and compare with previous results obtained with the same data using SPAC. Our data come from three experiments carried out in Parkway and Wainuiomata valleys in New Zealand, using broad-band portable stations. Interstation distances range from 5 m to 2.1 km, and our results are useful in the frequency band from 0.1 to 7 Hz. Frequency domain correlation requires an isotropic microtremor field, a condition that need not be satisfied in the time domain. Two station correlations provide useful results due to the temporal stationarity and isotropy, in average, of the microtremor wavefield. This manifests itself in the symmetry of the temporal correlation functions with respect to zero time. Our results show that the local velocity structure and the interstation distance are the key factors conditioning the frequency range where surface wave dispersion can be correctly measured either in frequency or time domains. We confirm that, when the interstation distance becomes much larger than the dominant wavelengths, only the correlation in time domain is useful. All our results indicate that the signal obtained in the correlation of vertical component microtremors is due to the fundamental mode of Rayleigh waves, which appears as the most stable propagation mode, without any indication of body waves.

Key words: cross-correlation, dispersion curves, Green's function, microtremors, SPAC, stationarity.

1 INTRODUCTION

Ground motion amplification studies require the shear velocity structure of the subsoil to predict seismic ground motion. Shear wave velocity profiles can also be used to classify sites based on the average shear wave velocity for the uppermost 30 m (commonly called the V_{30} value) of the soil column, an approach commonly used by engineers and building codes (e.g. Borcherdt 1994). Traditional seismic prospecting methods face important difficulties in this task. First, shear waves are difficult to generate using traditional sources. Second, in urban zones, they may be challenged by operational limitations, or be affected by large background noise. For these reasons, methods based on microtremor measurements have gained popularity.

Site effect studies based on microtremors have significant advantages. The measurements can be performed with ease, rapidly and at a low cost. Many studies have made recourse to the horizontal-to-vertical spectral ratios (e.g. Nakamura 1989; Lermo & Chávez-García 1994), which in certain cases can be used to estimate the local seismic amplification function. Array measurements of microtremors have also been largely used. For example, Horike (1985) and Kagawa (1996) used f–k spectra to determine phase velocity dispersion curves from array measurements of microtremors. We also observe many applications of the Spatial Autocorrelation (SPAC) method. This method, proposed by Aki (1957), allows the determination of the phase velocity dispersion curve at a site from microtremor measurements obtained with a circular array of seismographs. SPAC is based on the assumed stationarity and isotropy of the microtremor wavefield. Although in the original paper the possibility to analyse the horizontal components was described in detail, most applications have used only vertical component records, assuming Rayleigh waves. The possibility of determining several modes was proposed by Asten (1976, 1978).

In many applications of the SPAC method, the seismographs have been arranged on a half-circle (Ferrazzini *et al.* 1991; Metaxian 1994; Chouet *et al.* 1998), or have used at least three stations at the same distance from a central one recording simultaneously (Morikawa *et al.* 1998; Okada 2003; Apostolidis *et al.* 2004; Chávez-García *et al.* 2004; Roberts & Asten 2004). Ohori *et al.* (2002) applied the SPAC method using T-shaped arrays; however, they neglected to explain how they avoided the requirement to have an azimuthal average. A more radical approach has been that taken by Chávez-García *et al.* (2005) who showed that it is possible to use the SPAC method using data from single station pairs. The basis for this lies in the original paper by Aki (1957). If the microtremor wavefield is required to be isotropic and stationary both in time and space, then it is possible to replace the azimuthal average by a temporal one.

In the SPAC method, the data is analysed by computing cross-correlations in the frequency domain. Naturally, this operation can also be computed in time domain. As far back as 1968, Claerbout presented a scheme to produce reflection seismograms through the correlation of earthquake records. Baskir and Weller (1975), Nikolaev & Troitskiy (1987), Cole (1995) and Rickett (2001) showed that source–receiver type seismograms can be generated using cross-correlations of ambient seismic noise. Many recent papers have shown arguments, proofs and demonstrations that diffuse fields have time domain correlations closely related to the Green's function (e.g. Weaver & Lobkis 2001; Snieder 2004; Wapenaar 2004; Roux *et al.* 2005; Sabra *et al.* 2005a; Sanchez-Sesma & Campillo 2006). A brief historical review is given in the introduction section of Weaver & Lobkis (2005b). This relation has been observed in ultrasonics (e.g. Lobkis & Weaver 2001; Malcolm *et al.* 2004), acoustics (e.g. Sabra *et al.* 2005c), seismic coda (e.g. Campillo & Paul 2003; Paul *et al.* 2005) and ambient seismic noise (e.g. Shapiro *et al.* 2005; Sabra *et al.* 2005b). The reason behind the relation between cross-correlation and the medium's Green's function for all these different types of signals is that they share a property called modal equipartition of the wavefield (Campillo 2006; Sanchez-Sesma & Campillo 2006). This means that the wavefield consists of waves propagating in all directions and polarizations, with equal weight in average. The randomness of the coda results from multiple scattering on randomly distributed scatterers and it has been shown (Campillo 2006) that the diffusive regime for the coda emerges early, even if it only tends asymptotically to complete isotropy. Concerning ambient seismic noise, even if its actual origin is not fully known, we assume that it is the result of the summation of the effects of many sources, distributed randomly at the surface of the Earth. In fact, the development of the SPAC method (Aki 1957) required the assumption that the corresponding wavefield is stationary in both space and time. Aki (1957) observed that cross-correlation computed between stations aligned in different directions did not differ significantly, concluding that 'we may regard the microremor as being propagated in every direction, each with almost uniform power'. Thus, seismic noise and seismic coda are equivalent once the waves that form it have reached the diffusive regime. Campillo (2006) presents a complete summary of these results.

In a recent paper, Chávez-García & Luzón (2005) presented the analysis of ambient seismic noise recorded with a four station triangular array in Almería, Spain. They showed the equivalence between the SPAC method and the results obtained from the analysis of cross-correlations in time domain. However, their study was mainly focused on the determination of the wave types that were observed in the cross-correlation functions. Moreover, their study was strongly limited by the short duration of the microtremor windows analysed (5 min) and markedly by the fact that their data allowed them to compute cross-correlations for only two inter station distances (26 and 45 m).

In this paper, we pursue further the ideas presented in Chávez-García and Luzón (2005), bringing together data from three different experiments that were analysed previously in an independent way. Those previous analyses were limited to standard applications of the SPAC method to determine the subsoil structure. In this paper, we compute cross-correlations in the time domain and explore the empirical limitations associated with it in both domains. We use data from three experiments carried out in and around Parkway and Wainuiomata valleys in New Zealand. Our purpose in this paper is twofold. First, we analyse those data using cross-correlation in the time domain and compare the results with those that were obtained in the frequency domain using SPAC. Second, we want to consider together the results from the three experiments to determine the resolution limits of our data. Together, the data from three experiments made in the same region span an interstation distance range between 5 m and 2.1 km. We show that the local velocity structure and the interstation distance condition the frequency range for which the estimated phase velocities are reliable. Outside that range, if the interstation distance is too short, there is perfect correlation between the two stations and no information can be retrieved, even if theory predicts no such limitation. If that distance is too large, the wavelengths may be smaller than required by the basic sampling theorem, and the 2π ambiguity in the determination of the phase difference between stations renders useless the results in the frequency domain. In this case, however, time domain correlations may still be useful since it is still possible to measure group velocity as shown previously by Shapiro & Campillo (2004) and Chávez-García & Luzón (2005). We discuss the role of the isotropy of the microtremor wavefield in the symmetry of the results in time domain cross-correlation and in the validity of the SPAC results. Our experimental results are useful to better understand the correlation of real ambient noise records and their applicability in the determination of the local subsoil structure.

2 DATA

Our data come from three experiments carried out in and around the Parkway and Wainuiomata valleys in Northern Island, New Zealand during February of 2003. The Parkway and the Wainuiomata basins are small, shallow alluvial valleys located at the Southern tip of North Island, New Zealand (Fig. 1). Both basins share a common origin and consist of soft clays overlaying a weathered greywacke. Firm rock is only found about 2 km to the NW of Parkway basin. Geological studies at these basins include a stratigraphic drill hole (Begg *et al.* 1993), penetrometry tests (Barker 1996) and geophysical measurements. A summary of the results is given in Chávez-García *et al.* (1999).

The impedance contrast between the sediments and the greywacke is large in these basins, making for large local amplification (Chávez-García *et al.* 1999, 2002). For all three experiments we recorded ambient noise using five broadband, Guralp, CMG40 seismometers, coupled to Orion recorders (manufactured by Nanometrics). These data loggers have 24 bit A/D converters and each station included a GPS receiver, for precise time keeping. Although the records include the three components of motion, in this paper we will only deal with the vertical motion.

The first experiment had the five stations installed along a line at the centre of Parkway valley (Fig. 1). The experiment was described

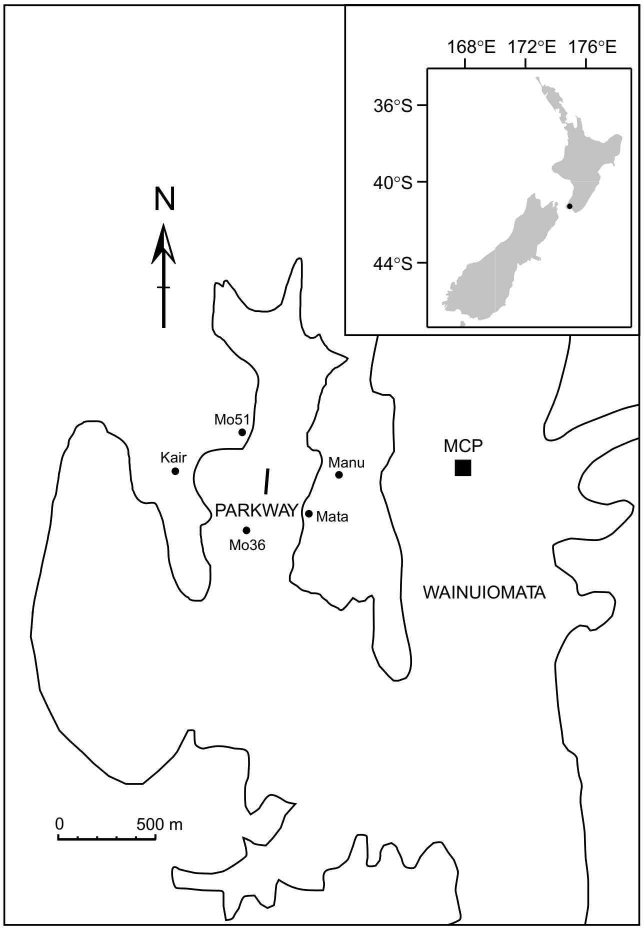

Figure 1. Parkway and Wainuiomata valleys in Northern Island, New Zealand. The solid line shows the boundary between soft rock (greywacke) exposure and sediment cover. The short, thick line within Parkway valley indicates the location of the linear array on the soft sediments. The solid square in Wainuiomata valley shows the location of the Mary Crowther Park (MCP) square array measurements on soft sediments. Five solid circles indicate the location of the stations for our third experiment, one on sediments (Mo36) and four on the soft rock surrounding Parkway valley (Mo51, Kair, Manu, Mata). The station at Mo36 had been installed first on hard rock, about 2 km to the NW of Parkway valley, at a location that falls outside the plotted area.

in Chávez-García *et al.* (2006), where the data were analysed in the frequency domain. The stations were first located with a spacing of 5 m and seismic noise was recorded for 30 min. The spacing between stations was then increased to 10 m, and another 30 min of ambient vibration were recorded. We increased subsequently the spacing between adjacent stations to 20 and 40 m, and each time half an hour of seismic noise was recorded. Unfortunately, one of the stations at the end of the array failed during the experiment, and we could use only the data from four stations from this experiment. The interstation distances span the range from 5 to 120 m. Chávez-García *et al.* (2006) computed correlation coefficients in the frequency domain as defined by Aki (1957), except for the azimuthal average. They inverted those coefficients using the standard procedure for the SPAC method to obtain a phase velocity dispersion curve, and then inverted that curve to obtain a soil profile. The dispersion curve that was obtained was considered to be reliable in the frequency band between 2.3 and at least 8 Hz.

The second experiment was carried out in the Northern part of Wainuiomata valley, at Mary Crowther Park (MCP in Fig. 1). This time, the five stations were arranged forming a square with a central station. The vertices of the square were located at 10 m from the centre. With this configuration, 35 min of ambient vibration were recorded. Then, the stations on the vertices were subsequently moved to a distance of 20, 40 and 80 m from the fixed central station, recording background noise for 35 min each time. Given the shape of the array, we obtained data for three different interstation distances from each array. For example, from the first array (10 m distance between the vertices and the centre of the square array) we obtained data for 10 m distance from four pairs, for 14.1 m from other four pairs, and for 20 m distance from the two pairs of stations at the opposite vertices. In Chávez-García *et al.* (2004), the data recorded at MCP was analysed again using standard SPAC, substituting temporal averaging for the azimuthal averaging. Chávez-García *et al.* (2004) computed phase velocity dispersion curves and obtained soil profiles from their inversion. They showed the good agreement that was obtained between the seismic response of the soil profiles for vertical shear wave incidence and the horizontal to vertical spectral ratios computed for the same ambient noise records. The results from the SPAC method were reliable between 2 and 4 Hz.

The third experiment was carried out around Parkway valley (Fig. 1). We occupied four sites on soft rock (weathered greywacke) with four of our broad-band stations. The fifth station was installed, first at a hard rock site (∼2 km to the NW of Parkway basin, outside the region plotted in Fig. 1) for 2 d, and then moved to the centre of the valley for an additional 2 d of continuous recording. The data was presented in Chávez-García *et al.* (2005). In that paper, SPAC's correlation coefficients were computed in the frequency domain and a phase velocity dispersion curve was inverted from them. However, that dispersion curve was intended for comparison with a previous one obtained for the same site using ambient noise records obtained with an array of 1 Hz sensors. For that reason, Chávez-García *et al.* (2005) selected 1-min long windows for the analysis. The inversion of the correlation coefficients allowed them to determine a phase velocity dispersion curve that was considered reliable in the frequency range from 0.4 to 1.4 Hz. The lower frequency limit could be due to the choice of 60-s time windows of data. In this paper, we wanted to explore the limits of the method, and we repeated the analysis using the SPAC method using longer time windows of data in the analysis. In an attempt to estimate correlation coefficients at smaller frequencies, we selected 100 20-min non-overlapping windows from the original records. Unfortunately, the results showed that we had not been careful enough during the installation of the stations. The sensors were not well levelled and the records suffer from tilt-induced noise that does not allow to interpret the results for frequencies below 0.1 Hz. The dispersion curve obtained using 20-min windows does not differ significantly from that presented in Chávez-García *et al.* (2005).

3 CORRELATION RESULTS IN THE TIME DOMAIN

Consider the computation of the correlation between two simultaneous traces of ambient noise computed in the time domain. In recent years, many papers have been published showing the relation between the cross-correlation function of a diffusive signal at two receivers and the time domain Green's function of the medium between the recording sites. The cross-correlation of ambient seismic noise recorded at two locations can be written as (Sabra *et al.* 2005b)

$$C_{ij}(\tau) = \int_0^T \upsilon_i(r_1, t)\upsilon_j(r_2, t+\tau)\mathrm{d}t, \qquad (1)$$

where t is time, $\upsilon_i(r_1, t)$ and $\upsilon_j(r_2, t)$ are ambient noise windows recorded at locations r_1 and r_2, T is the observation period and C_{ij} is the cross-correlation computed between the two traces as a function of τ, the delay time. Sabra *et al.* (2005b) proposed

$$\frac{\mathrm{d}C_{ij}}{\mathrm{d}t} \approx -G_{ij}(r_1; r_2, t) + G_{ji}(r_1; r_2, -t), \tag{2}$$

where $G_{ij}(r_1; r_2, t)$ is the time domain Green's function between the two locations. Eq. (2) predicts an asymmetrical shape with respect to origin time. In our analysis, we will not interpret the Green's functions and, for that reason we have not differentiated our correlation functions with respect to time (see Sabra *et al.* 2005b, for a comparison between the cross-correlation functions and their derivatives). Thus, our correlation functions should be symmetric with respect to origin time, if reciprocity applies or (an equivalent statement) if the wavefield propagates with equal power in the two directions r_1r_2 and r_2r_1.

We deal first with the results for the linear array at Parkway basin. We selected 60 s windows (with 20 s overlap), chosen from the 30 min of data recorded for each interstation distance. Each simultaneous pair of records was baseline corrected, cosine tapered over 10 per cent of its length and the normalized time correlation between the signals was computed using a hamming smoothing. We averaged all cross-correlation functions computed for station pairs at the same interstation distance. Fig. 2 shows the results, as a seismic section. Each averaged cross-correlation function is shown at the corresponding interstation distance, and the traces were bandpass filtered between 0.5 and 10 Hz. Fig. 2 shows a very clear pulse, symmetric with respect to zero time, appearing at increasing time with increasing distance. The signal-to-noise ratio (SNR) of this pulse decreases with increasing distance. Its phase velocity is between 250 and 300 m s^{-1}. When the interstation distance is much smaller than the dominant wavelength, we expect the cross-correlation function to degenerate to the autocorrelation function. The trace plotted at 5 m distance shows a single pulse that is symmetric about zero time, as expected for wavelengths between 50 and 150 m.

Fig. 3 shows a similar result for the ambient noise records obtained at MCP. In this case, we used non-overlapping, 60 s windows. Again, a pulse, symmetrical with respect to zero time, appears at increasing times with increasing distance. The symmetry of the pulse suggests once more that no preferred direction of propagation exists. The

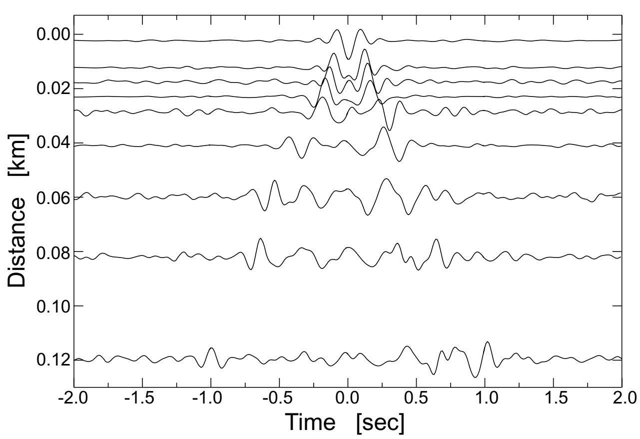

Figure 2. Seismic section formed with the correlation functions computed in time domain for all station pairs from the linear arrays at Parkway. Each trace is plotted at the corresponding interstation distance and is the average for all station pairs at that interstation distance. We observe a clear pulse, symmetric with respect to zero time.

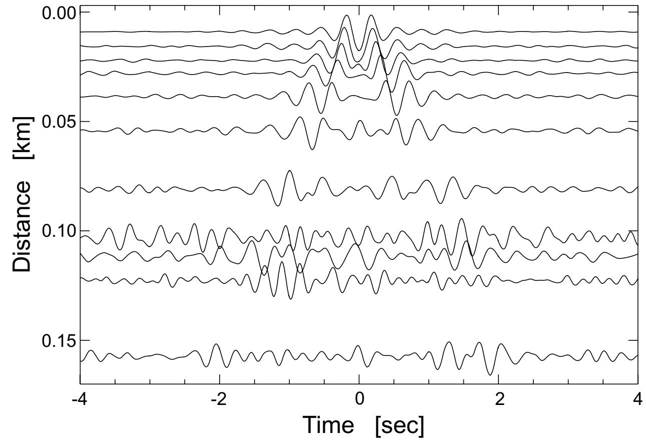

Figure 3. Seismic section formed with the correlation functions computed in time domain for all station pairs from the square arrays at the MCP site. Each trace is plotted at the corresponding interstation distance and is the average computed for all station pairs at that interstation distance. We observe a clear pulse, symmetric with respect to zero time.

presence of the causal (retarded) and anticausal (advanced) Green's functions expected theoretically is an evidence of the isotropy of the microtremor wavefield (Paul *et al.* 2005). The apparent velocity in the seismic section is smaller than that observed in Fig. 2, and the frequency content of the pulse is shifted towards lower frequencies. This was expected, given the smaller resonant frequency observed for MCP relative to Parkway valley. The pulse is clearly observed for small distances. For larger distances, its SNR decreases until it disappears for distances larger than 80 m. The array used at MCP (a square with a fifth station at its centre) allows investigating further the symmetry of the observed pulse with respect to zero time. Fig. 4(b) shows the average correlation in time domain computed for each station pair separated a distance of 40 m. The trace corresponding to 40 m in Fig. 3 is the average of the traces shown in Fig. 4(b). Traces 1 and 3 in Fig. 4(b) show a very clear symmetry, which is not the case for traces 2, 4 and 5 (the sixth trace has a low SNR). Traces 2, 4 and 5 were computed for station pairs oriented along an azimuth of 165° (Fig. 4a), that is, almost north–south, while traces 1 and 3 correspond to station pairs oriented along the azimuth 75°. Thus, the correlation between stations pairs aligned in a north–south direction suggests that the microtremor wavefield has a dominant direction of propagation towards the south (this direction is consistent for the three station pairs in the north–south direction). Station pairs aligned in an almost east–west direction show correlation functions that are symmetric with respect to zero time, indicating that there is similar power in the waves travelling to the east as in the waves travelling to the west, but the SNR is smaller than that for station pairs aligned almost north–south. A significant point is that, in spite of these differences, the time at which a pulse occurs does not depend on the orientation of the station pair, as would be the case for ballistic waves. This weak anisotropy of the wavefield is lost when we average them all together to obtain the mean cross-correlation function for 40 m interstation distance shown in Fig. 3.

Consider now the results for the third experiment, where we have merged the results from its two different stages. We used again the 100 20-min windows, mentioned in the data section. The results are shown in Fig. 5. This time, the pulse that appears in the correlation between station pairs has very low frequency (around 0.6 Hz). The traces for distances smaller than 700 m correspond to pairs of

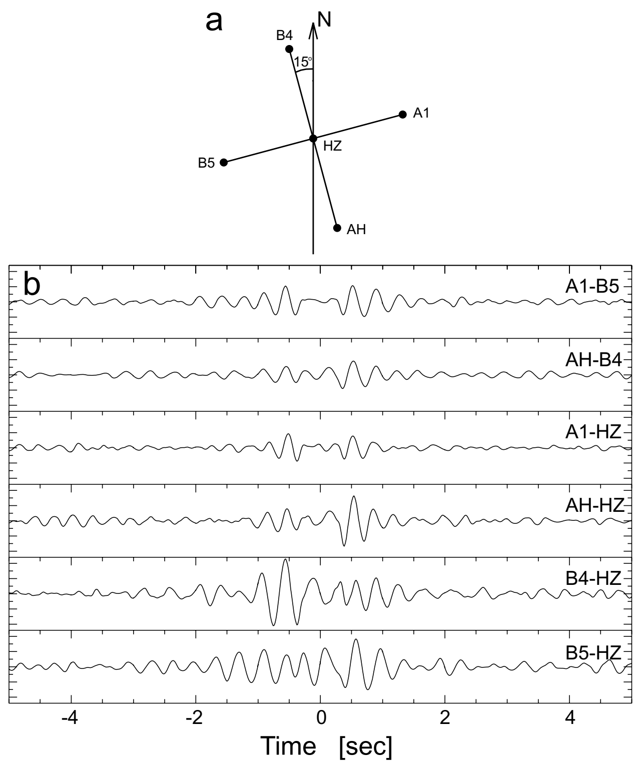

Figure 4. (a) Schema of the station distribution for the square arrays at MCP. (b) Correlations in time domain for station pairs separated a distance of 40 m computed using records from the MCP site. Each trace is the average for all windows analysed for a particular station pair. The first two traces come from the array with 20 m distance between the corners and the central station. The last four traces come from the array with 40 m distance between the corners and the central station. The average of the six traces shown here was plotted at 40 m distance in the previous figure. The amplitude scale is arbitrary, but common to all traces.

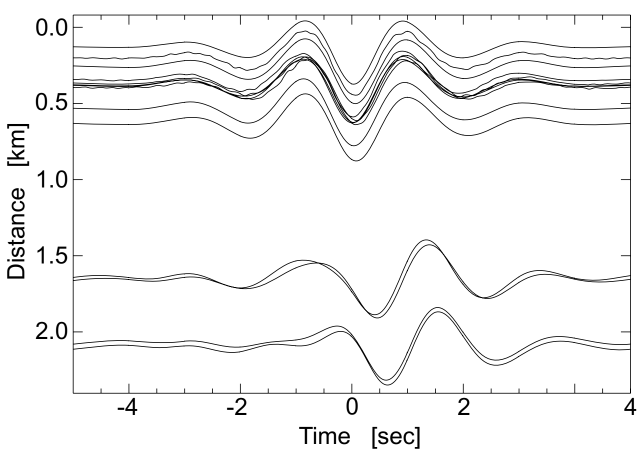

Figure 5. Seismic section formed with the correlation functions computed in time domain for all station pairs from the third experiment. Each trace is plotted at the corresponding interstation distance and is the average computed for all station pairs at that interstation distance.

stations that are too close relative to wavelength (estimated to be between 4 and 26 km) to show any significant difference from zero phase. We verified that those traces cannot be distinguished from the autocorrelation functions computed for each station. The traces at distances larger than 1.5 km show a clear pulse that is not symmetric with respect to zero time, and suggest that the microtremor wavefield has a dominant direction of propagation to the south. Phase velocity in Fig. 5 is about 3200 m s^{-1}.

Fig. 6 shows Fourier amplitude spectra for a representative example of the cross-correlation between station pairs from the three experiments. The correlation pulse observed for the linear array includes energy in the frequency range between 2 and 7 Hz. In contrast, the pulse observed at MCP is above noise in the frequency band from 1.2 to 4 or 5 Hz. Finally, the third experiment shows that the pulse in the time domain cross-correlations is above noise between 0.12 and 0.8 Hz.

4 DISCUSSION

4.1 Frequency limits of validity in time and frequency domains

The frequency range of validity of the SPAC method was established in terms of the relation between the interstation distance and the wavelength by Henstridge (1979). This author recognized that the correlation coefficients could be considered reliable in the range

$$2 \le \frac{\lambda}{r} \le 15.7 \qquad (3)$$

where r is the interstation distance and λ is the wavelength. The limit $r \ge \lambda/15.7$ constrains the validity of the results at low frequencies. When the wavelength becomes too large with respect to the interstation distance, the phase difference between stations becomes negligible and no useful information can be derived. The limit $r \le \lambda/2$ represents the spatial version of the fundamental sampling theorem, limiting the high frequency end of the results. When the data were analysed in the frequency domain, it was checked that the phase velocity dispersion curve obtained in each case obeyed in a general sense the limits established by eq. (3). However, it was noted that the value of r that applied was not the largest nor the smallest interstation distance that was included. Eq. (3) was obtained for a single distance r, and it cannot be straightforwardly applied to the case where many distances are available for analysis.

The limits of eq. (3) also apply to the results of the correlation computation in the time domain. There is, however, a very important difference between time and frequency domains in this respect. The SPAC method establishes a formal relation between the correlation coefficients and the phase velocity dispersion curve. The computation of the correlation in time domain, in contrast, allows estimating the Green's function of the medium. This Green's function can be used to recover information on the phase or group velocities (e.g. Chávez-García & Luzón 2005). The estimates of phase velocity need to obey the limits established by Henstridge (1979). We checked this for our data. However, if we use the temporal correlation of noise records to estimate group velocity dispersion, it is only necessary to identify a surface wave group in the correlation function and compute group velocity dispersion using, for example, the multiple filter technique (Dziewonsky *et al.* 1969). This is the reason behind the success of Shapiro and Campillo (2004) and Shapiro *et al.* (2005), when they estimated group velocity values at two periods for station pairs separated a distance $r \gg \lambda$.

4.2 Dominant directions in the microtremor wavefield

Sabra *et al.* (2005b) showed that, in a very consistent way, the largest SNR for their temporal correlation functions was obtained between pairs of stations aligned perpendicularly to the Pacific Ocean coast

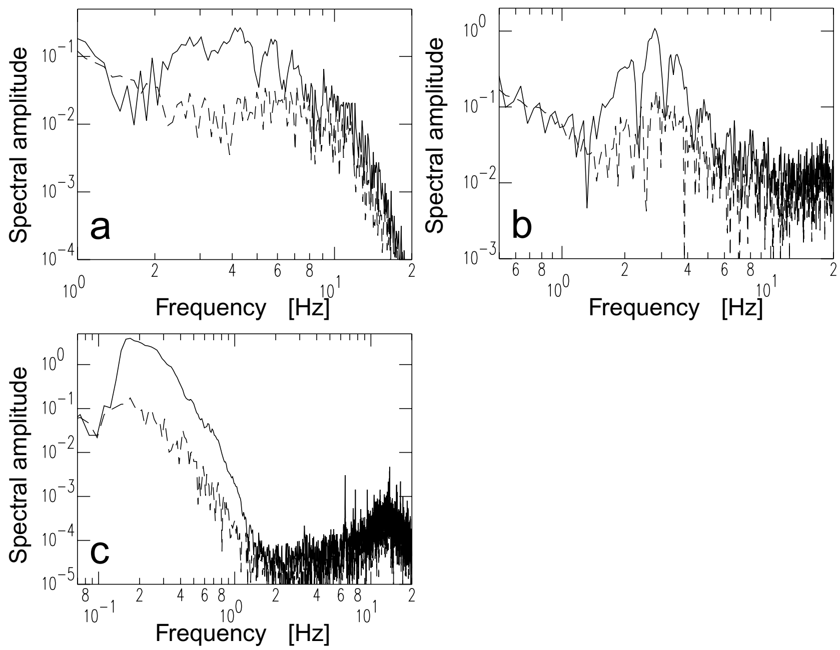

Figure 6. Solid lines: Fourier amplitude spectra of the time window including the significant pulse observed in a representative correlation function for each one of our three experiments. Dashed lines: Fourier amplitude spectra of a time window outside the significant pulse for the corresponding correlation function. (a) Spectra computed using 10 s windows from the average correlation function observed at 40 m interstation distance for the linear arrays at Parkway valley. (b) Spectra computed using 14 s windows from the average correlation function observed at 40 m interstation distance for the square arrays at MCP. (c) Spectra computed using 60 s windows from the average correlation function observed at 2116 m interstation distance for the third experiment.

in Southern California. Conversely, the smaller SNR were observed for station pairs oriented parallel to the coast. They interpreted this as the consequence of the preferred propagation direction for a microtremor wavefield generated by the action of the waves on the coast. The symmetry of the time domain correlation functions is discussed in detail in Paul *et al.* (2005). They show that this symmetry is a measure of the isotropy of the wavefield; perfect isotropy implies that the amplitude and shape of the retarded (causal) and advanced (anticausal) Green's functions are the same.

Figs 2 and 3 show symmetric seismic sections with respect to zero time for the linear array and MCP. However, when we looked in detail, we observed a lack of symmetry at the MCP site, with a dominant direction of propagation from north to south. Our results coincide with those of Sabra *et al.* (2005b) in that SNR is larger when the station pair analysed is oriented in the preferred direction of propagation. This preferred direction is lost when we average all station pairs at the same interstation distance. The SNR depends on the homogeneity of the scattered wavefield, as shown by Derode *et al.* (2003) and Paul *et al.* (2005). SNR scales also with the square root of the recording time (Sabra *et al.* 2005b) and with frequency (Weaver & Lobkis 2005a). A dominant propagation direction for ambient seismic noise may result from the location of the source of the ambient vibration, as seems to be the case for Sabra *et al.* (2005b) and Paul *et al.* (2005). However, it could also be related to the influence of the geological structure where the measurements are made. In the case of Parkway basin, Chávez-García *et al.* (2002) observed that earthquakes recorded at Parkway basin excited diffracted surface waves preferably propagating to the South. We may speculate that Wainuiomata basin, being similar to Parkway, might behave in the same way, and the preferential southward direction observed at the MCP site be the preferred direction of motion in this valley.

Fig. 5 shows a seismic section where only the causal Green's functions appear. According to the papers cited, this implies an anisotropic wavefield. However, contrary to the case of MCP, we could not average cross-correlations along paths with an orientation different from the NW. Thus, we are unable to tell whether averaging cross-correlations along a different direction would result in symmetric Green's functions.

An important final point concerns the comparison between time and frequency. The SPAC method requires isotropic ambient noise, whereas our results in time domain show the existence of a preferred direction of propagation in the cases of MCP and the third experiment. However, when we analysed those data using the SPAC method, we obtained very good results (Chávez-García *et al.* 2004, 2005). We conclude retrospectively that, even if theory requires perfectly isotropic noise for the SPAC method to work, useful results can be obtained even if this requirement is not rigorously satisfied.

4.3 The frequency band where the cross-correlation provides results

The frequency band where the pulses appeared in the correlation functions in time domain is different for each experiment and depends on the wavelengths that can be correlated between stations. In this way, the shear wave velocity profile at a site determines the interstation distances at which results can be obtained from the cross-correlation of microtremors. All our observations indicate that what we observe are the fundamental modes of Rayleigh waves, which seems to be the most stable propagation mode. Fig. 7 shows the dispersion curves for the fundamental mode of Rayleigh waves at the experiment sites (Chávez-García *et al.* 2004, 2005). We have indicated on this figure the frequency range for which we obtained useful results from each of our three experiments. In all three cases, the phase velocities that can be determined through the inversion of the correlation coefficients, or those that can be measured from the

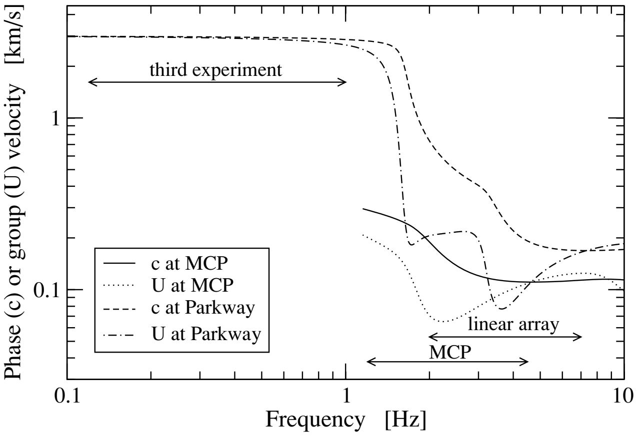

Figure 7. Phase (c) or group (U) velocity dispersion curves for the fundamental modes of Rayleigh waves at Parkway and MCP sites. They were obtained from previous analysis at these sites. The horizontal lines with arrows indicate the frequency range for which the results derived from the correlation analysis yield useful information.

seismic sections built with the temporal correlation functions, coincide with the dispersion curves shown in Fig. 7. The same applies to the group velocities that can be measured from the temporal correlation functions. Thus, the estimate of the Green's functions obtained from our temporal correlation functions is limited to a narrow frequency band, and corresponds exclusively to surface waves.

5 CONCLUSIONS

We have presented an analysis of correlation between microtremors recorded at station pairs using temporal, broad-band stations. Our analysis was carried out in the time domain and brought together data analysed previously in the frequency domain in an independent way. We have used data from three experiments carried out in and around Parkway and Wainuiomata valleys in New Zealand. Our data sets span the distance range from 5 to 2116 m, and we obtain useful results in the frequency band from 0.1 to 7 Hz.

We showed that the local velocity structure and the interstation distances condition the frequency range for which the estimated phase velocities are reliable. We showed also that, within that range, correlation in frequency or time domains give equivalent results. This equivalence is expected theoretically but has seldom been explicitly considered and still requires to be demonstrated empirically. In addition, theoretical papers relate time domain correlations to the complete Green's function of the medium. Our results show that in the real world the retrieval of Green's function is not complete and has the same frequency band limitations as the SPAC method. Outside that frequency band, if the interstation distance is too short relative to wavelength, there is perfect correlation between the two stations and no phase-difference information can be retrieved, even if theoretically we would expect to recover body wave information. In the time domain, the corresponding cross-correlation functions degenerate to the autocorrelation functions. If the interstation distance relative to wavelength is too large, the wavelengths may be smaller than required by the basic sampling theorem, and the 2π ambiguity in the determination of the phase difference renders useless the results in the frequency domain. In this case, however, time domain correlations may still be useful since it is possible to measure group velocity, as has been shown previously by Campillo and Paul (2003), Shapiro and Campillo (2004) and Chávez-García and Luzón (2005).

Isotropy of the microtremor wavefield leads to cross-correlation functions in time domain that are symmetric with respect to zero time. Our time domain correlations for the first and second experiments showed clear symmetry, in average, with respect to origin time. This indicates that the microtremor wavefield is, in average, isotropic, with no preferred direction of propagation. In detail, we observed anisotropy at the MCP site and for the third experiment. The information on a dominant direction of propagation is lost when we average cross-correlations for station pairs oriented along different directions at the same interstation distance. We could not test this for the third experiment. The observed anisotropy for the wavefield contradicts the requirements for the SPAC method to be useful (Aki 1957). However, we had previously obtained good results using the SPAC method for all three experiments. We conclude retrospectively that the isotropy requirement in SPAC need not be rigorously satisfied, and that it may still be possible to obtain a valid average in the frequency domain, even when ambient noise is not perfectly isotropic. Our empirical results thus confirm the numerical experiments presented in Paul *et al.* (2005).

All our observations indicate that what we observe is the fundamental mode of surface waves (Rayleigh for our vertical component records) in agreement with previous results, and no indication of body waves in contradiction with theoretical expectations. Surface waves are a very stable mode of propagation because they are less sensitive than body waves to lateral heterogeneities. The frequency range of the results is limited to a narrow frequency band for each interstation distance and thus the information that can be retrieved from the medium will be likewise limited. This means that we are not necessarily looking at the same microtremor wavefield at short and long interstation distances. The use of many interstation distances at a given site can help to solve this limitation.

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