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Research Article

Dynamic Matching in Cloud Manufacturing considering Matching Costs

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As a service-oriented business platform model, the nature of cloud manufacturing is to realise the manufacturing resources' sharing, which will largely benefit resources supplier, resources demander, and platform operator. However, it also faces some new problems. One of the most critical issues is how to dynamically match resources of supply and demand to maximise profits of all parties while considering matching costs. This paper investigates the resources' dynamic matching in a manufacturing supply chain that operates under a cost-sharing contract and consists of two independent and competing manufacturers and a resource-service platform. We first use differential equation to model the evolution of resource-sharing and capture the effect of matching service efforts on market demand. Next, we study the optimal matching strategies by a two-stage differential game based on the dynamic control approach. Then, we design a cost-sharing contract to coordinate and improve the supply chain's performance. Finally, a numerical example is provided to illustrate the impact of platform transaction fees and matching costs on the feasible region of the corresponding contract.

1. Introduction

Information technologies, such as the Internet of Things, cloud computing, and cyber-physical systems, impact daily life through their powerful data-processing capacities. For example, e-commerce has become an indispensable means of shopping over the past two decades, and a small number of giant e-commerce companies, such as Amazon, eBay, and Alibaba, have emerged to dominate the market. However, the Internet also enables other types of transaction, such as sharing. Online networks facilitate the sharing of computing and manufacturing resources in supply chain. This has resulted in collaborative consumption and collaborative production: peer-to-peer exchanges for obtaining, providing, or sharing access to goods and services, facilitated by community-based online platforms. The manufacturing industry is undergoing a major transformation enabled by cloud computing. The main thrust of cloud computing is to provide on-demand computing services with high reliability, scalability, and availability in a distributed environment. Cloud technologies have had profound impacts on production management in manufacturing [1]. O'Rourke [2] stated that new information technologies have helped to drive the development of 'lean manufacturing', with factories using such technologies being better equipped to rapidly deliver the products that customers want. As information technologies become embedded in all aspects of production, 'network-centric' manufacturing advances throughout value chains and each element becomes 'smart', thereby optimising efficiency throughout a product's life-cycle [3].

Learning from cloud computing, researchers have proposed a model of 'cloud manufacturing', in which uniform manufacturing resources are shared through online networking. In this model, manufacturing capabilities and resources are shared via a cloud platform. The status of idle resources is updated and released in real time to facilitate online transactions and identify the most sustainable and robust manufacturing route possible [4]. Figure 1 presents a simplified model of the common features of cloud manufacturing. The cloud manufacturing architecture defines three common roles (although the exact nomenclature for each role varies in the literature): the supplier (which offers services or resources on the platform), the demander (which requests services or resources through the cloud), and the

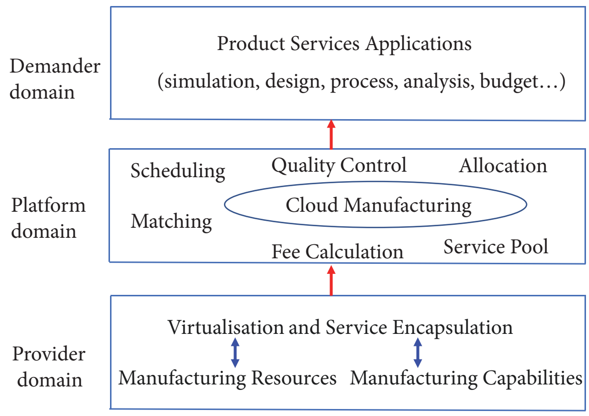

Figure 1: Cloud manufacturing architecture, adapted from [1, 5, 9].

platform manager [5–7]. The demander utilises resources or services for manufacturing purposes and the supplier provides these resources or services by renting, leasing or lending equipment or other resources for short-term periods. The cloud platform manages the use, performance, and delivery of services and negotiates the relationship between supply and demand; it acts as an intermediary, providing connectivity and transport to enable the exchange of services between consumers and providers [8]. In this regard, cloud manufacturing and e-commerce share some similarities, the main difference being that commodities are traded on an e-commerce platform whereas manufacturing services are exchanged on a manufacturing platform. In the reallocation process, idle manufacturing resources and capabilities are connected through cloud computing and other information technologies, eventually forming a supply chain for manufacturing resource sharing.

In China, Shandong province has built an ‘industrial cloud platform’ that incorporates regional factories and technological resources. Factories and individuals can request various services and resources through the platform at low rental costs. The platform has already had economic and environmental benefits [4]. The transition from traditional manufacturing to a service-oriented model occurred gradually, as lessons from the ‘sharing economy’ were adapted to the manufacturing sector. Unlike in the sharing economy, in which users share consumer products, the platform facilitates the sharing of idle manufacturing resources. A number of platforms have already implemented business models that closely resemble cloud manufacturing. For example, MFG.com, the world’s largest contract manufacturing marketplace, provides a fast and efficient platform for exchanging manufacturing resources. Similarly, 1688.com, China’s leading e-commerce platform for domestic small enterprise trading, adopted similar strategies for sharing manufacturing resources. As of 2018, 1688.com’s business model covered 16 industries and a wide range of supply services, from raw materials to industrial products, clothing, apparel, and household items. Manufacturing resource sharing has obvious benefits for resources supplier, resources demander and platform operator. However, it also introduces new management challenges. One of the most critical issues is optimising the dynamic matching of supply and demand to maximise cooperation between the various parties while considering matching costs.

The goal of matching is to connect consumer demand to the right products or services. To improve matching, all parties in the supply chain (supplier, demander, and platform manager) must invest in the matching effort. As Figure 1 shows, each party in the cloud manufacturing system incurs a distinct set of matching costs [9, 10]: (i) the supplier (the resource or service provider) incurs service-realisation costs, i.e., the cost of updating the platform to reflect the current status (availability and quality) of the resources, services, and capabilities; (ii) the platform manager incurs aggregation and generation costs, i.e., the costs of computing, storage, and scheduling; and (iii) the platform demander incurs invocation costs related to business operations, i.e., consultation, market analysis and investigation, purchase, insurance, etc. Optimising the allocation of resources and services for the supply chain is complex because it requires ensuring that the supplier, demander, and platform manager each benefit. In the process of reallocating supply chain resources, how to integrate, share, and optimise the allocation of supply chain resources so that the resources provider, cloud platform, and resources consumers can get the greatest benefits is an important issue faced by supply chain enterprises. The aim of this paper is therefore to identify matching strategies that can achieve this optimal solution.

There has been extensive research on performance analysis and supply-demand matching for manufacturing resources and services. In cloud manufacturing, operators use searching and matching algorithms to find suitable services to satisfy users’ requests. Several resource-service discovery frameworks are described in the literature. Tao et al. [11] proposed a four-phase method for resource-service matching and searching on service-oriented manufacturing system platforms. A genetic algorithm based model to search for the result that best matches a customer’s request is proposed in Zhang et al. [12]. Based on grey correlation theory, a machine tool supply-demand matching method is proposed in Xiao et al. [13]. Wang [14] investigated the cloud manufacturing resource discovery mechanism and proposed a manufacturing resource discovery framework based on the Semantic Web. Capturing user requirements and cloud services matching are important steps for realising on-demand resource-service provision that require the semantic description of manufacturing tasks. Wang et al. [15] investigated the semantic modelling and description of manufacturing tasks in cloud manufacturing system for manufacturing task to be better to match with manufacturing services. Li et al. [16] proposed a multilevel intelligent matching method to realise rapid, efficient and accurate matching. Yin et al. [17] proposed an input, output, precondition, effect matching model based on Web Ontology Language for cloud manufacturing. The model’s matching process is divided into three phases: parameter matching, attribute matching, and comprehensive matching. Li et al. [18] proposed an intelligent service searching and matching method of cloud manufacturing according to service type and state information.

The abovementioned studies have mainly examined issues of matching and scheduling with static manufacturing tasks and static candidate resource services in a given period. The dynamic changes typical of the practical process of supply-demand matching and scheduling have not been considered. Cheng et al. [19] proposed a supply-demand matching hypernetwork of manufacturing services, comprising a manufacturing service network, a manufacturing task network, and hyperedges between those two networks. Subsequently, based on the results in [19], Cheng et al. [20] formulated a model for revealing the matchable correlations between each service (supply) and each task (demand), subject to dynamic demand. Cloud manufacturing systems contain many dynamic elements. The number of users and the number of manufacturing tasks change dynamically. Additionally, in an environment of distributed resources, the relative independence of various economic entities also leads to dynamic changes in the sharing relationship. Cheng at al [19, 20] only consider the dynamic complexity caused by changes in the numbers of users and manufacturing tasks. They analysed supply-demand matching in cloud manufacturing from a technical perspective but neglected operations management concerns. From the latter perspective, the goal of matching is to connect consumer demand to the right products or services. This generally involves facilitating information exchange between a supplier and a demander. As matching becomes more successful, sharing increases. To improve matching, all parties in the supply chain (supplier, demander, and platform manager) must invest in the matching effort. However, this investment becomes an issue as the platform's matching abilities improve. Crucially, when the number of sharing transactions on the platform increases, the matching costs also increase. Matching costs have not been considered in previous studies. Thus, our study has an important difference from the abovementioned studies, which is that we investigate the complex relations and conflicts of interest arising from the sharing of resources through cloud manufacturing from the perspective of operations management.

Game theory is a powerful theoretical tool for analysing conflict and cooperation behaviour among rational individuals and, as such, can be useful for optimising manufacturing resource sharing and management, from locating services and supply-demand matching to transactions [21]. Games can be either cooperative or noncooperative depending on whether parties share a formal agreement. Enterprises have variously competitive and cooperative relationships, depending on the functional dependency of their services or products. The service composition in cloud manufacturing should therefore ensure the functional realisation of composite services while guaranteeing that each enterprise profits. Game theory is uniquely suited to this type of problem. The key to apply game theory in service composition in cloud manufacturing is to design proper utility functions for each enterprise by comprehensively considering their service attributes (including economic attributes) and constructing gaming models or mechanisms for the appropriate service interactions [22]. Differential games offer a promising approach. For example, De Giovanni [23] and Amrouche et al. [24] developed differential game models to incorporate channel dynamics. Here, we use differential equations to model the dynamic evolution of manufacturing resource sharing and capture the effect of matching efforts on market demand. By applying optimal control theory, we derive matching strategies for both centralised and decentralised systems. We also design a cost-sharing contract to improve the performance of the decentralised supply chain. Finally, we use a numerical example to examine the feasibility and efficacy of platform transaction fees and other parameters as strategies for optimising the coordination contract. The paper makes three primary contributions, which can be summarised as follows. First, we investigate operational problems for a sharing supply chain from a dynamic matching perspective. Second, we design a coordination contract for the supply chain by accounting for the impact of resource-sharing levels, which can be used to coordinate the decentralised system in dynamic environments. Finally, to the best of our knowledge, our study is the first to explore supply-demand matching issues by applying optimal control theory and game theory to derive optimal solutions.

The study proceeds in six sections. In Section 2, we give descriptions of the notations and assumptions used throughout the paper. Section 3 provides the theoretical results for the optimal strategies under a decentralised decision scenario, Section 4 provides the theoretical results for the optimal strategies under a centralised decision scenario, and Section 5 provides the theoretical results for the optimal strategies under a coordination-contract scenario. The numerical results and sensitivity analyses are represented in Section 6. Finally, Section 7 concludes the study and discusses its implications for management.

2. Problem Description and the Basic Model

2.1. Problem Formulation. We consider a supply chain formed of two independent and competing manufacturers, labelled d and s, and a resource-service platform, labelled p, in which manufacturer s (i.e., the supplier) has surplus manufacturing resources, whereas manufacturer d (i.e., the demander) lacks such resources. The platform has a strong reputation and the supplier sells its manufacturing resources to the demander through the platform. Ultimately, the two manufacturers produce homogeneous products and sell them to consumers. Deciding the optimal efforts for matching to enhance sharing is the primary objective of the players, which wish to increase demand and subsequently profits by adopting the optimal operational strategies. A simplified channel structure of the sharing supply chain is presented in Figure 2.

Table 1 provides the notation used throughout the study.

Supply-demand matching within the supply chain is a complex issue. The level at which manufacturing resources are shared within a dynamic framework can be investigated using the following equation:

$$\begin{aligned} R(t)' &= \left\{\alpha A_s(t) + \beta A_p(t) + \gamma A_d(t)\right\} - \varphi R(t), \\ R(0) &= R_0 \geq 0, \end{aligned} \tag{1}$$

Table 1: Notation and descriptions.

Variable	
$A_s(t)$	matching effort of supplier
$A_d(t)$	matching effort of demander
$A_p(t)$	matching effort of platform
$R(t)$	manufacturing resources sharing level
ε	the platform's support rate
V	the value function
J	profit
Parameter	
π_s	the margin profit of supplier
π_d	the margin profit of demander
c	fees from demander to platform
ω	purchasing cost of demander
ρ	discount rate

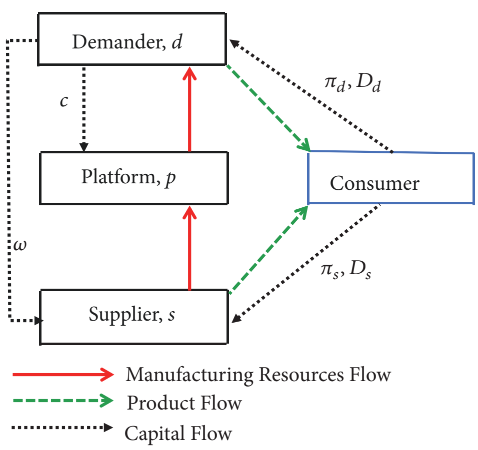

Figure 2: Channel structure of the sharing supply chain.

where α, β, and γ represent the marginal contribution of matching efforts to the sharing level, which we call *matching effectiveness*, and φ is the sharing level's decay rate. Matching effectiveness captures the relationship between each supply chain member's investment in matching and the sharing level. The sharing level's decay rate might reflect several scenarios; for instance, it could suggest that a manufacturing resource needs to be improved due to an increase in the number of product categories or attributes, which in turn can result in a decrease in the sharing level over time.

Each supply chain member's matching costs are convex and increasing, indicating that the matching efforts' marginal costs increase and are assumed to be quadratic:

$$C(A_s(t)) = \frac{\mu_s}{2} A_s^2(t),$$
$$C(A_p(t)) = \frac{\mu_p}{2} A_p^2(t), \quad (2)$$
$$C(A_d(t)) = \frac{\mu_d}{2} A_d^2(t),$$

where μ_s, μ_d, and μ_p are the positive cost parameters. This cost function is commonly applied in the literature [23–25].

The level of manufacturing resource sharing has a positive external spill-over effect on the supply chain's supplier and demander. Customer demand depends on both the marginal profit and the level at which manufacturing resources are being shared (i.e., the sharing level). The demand functions can be expressed as follows:

$$D_s(R(t), t) = a - \pi_s + \theta(\pi_d - \pi_s) + \eta_s R(t), \quad (3)$$

$$D_d(R(t), t) = a - \pi_d + \theta(\pi_s - \pi_d) + \eta_d R(t), \quad (4)$$

where a represents the potential market size, $\theta > 0$ denotes cross-price sensitivity between the two manufacturers, and $\eta > 0$ represents the effects of the sharing level on market demand. This is similar to the demand functions used in [24, 26–30], which depict the substitution effect between two independent and competing manufacturers.

2.2. The Objective Function. Assuming an infinite time horizon and a positive discount rateρ, the objective functions are

$$J_s = \max_{A_s} \int_0^\infty e^{-\rho t} \{\pi_s D_s(t) - C(A_s(t)) + \omega R(t)\}\, dt, \quad (5)$$

$$J_p = \max_{A_p} \int_0^\infty e^{-\rho t} \{cR(t) - C(A_p(t))\}\, dt, \quad (6)$$

$$J_d = \max_{A_d} \int_0^\infty e^{-\rho t} \{\pi_d D_d(t) - C(A_d(t)) - (\omega + c) R(t)\}\, dt. \quad (7)$$

To recapitulate, (1), (5), (6), and (7) define a differential game with three players, three control variables $A_s(t)$, $A_d(t)$, and $A_p(t)$, and one state variable $R(t)$. The controls are constrained by $A_s(t) \geq 0$, $A_d(t) \geq 0$, and $A_p(t) \geq 0$. The state constraint $R(t) \geq 0$ is automatically satisfied. We assume that the game is played à la Stackelberg, with the platform acting as the leader and the two manufacturers as followers (see [24, 25] for examples of the Stackelberg differential game).

3. The Optimal Strategies in the Decentralised System

We start by analysing the first scenario, in which the players implement a noncooperative program. Under decentralised decision-making, the supplier, platform, and demander maximise their own profits, respectively. The platform is the channel leader and does not offer subsidies to the demander. We use the superscript 'N' to signify the decentralised system scenario.

The supply chain game can be conceptualised in two stages. In the first stage, the platform decides the matching efforts $A_p(t)$. In the second, both the supplier and demander make their decisions, respectively. In particular, the supplier determines the matching efforts $A_s(t)$ and the demander determines the matching efforts $A_d(t)$. The sequence of the events is shown in Figure 3.

From this point forward, the time argument is omitted. Let V_s^N, V_p^N, and V_d^N denote the players' value

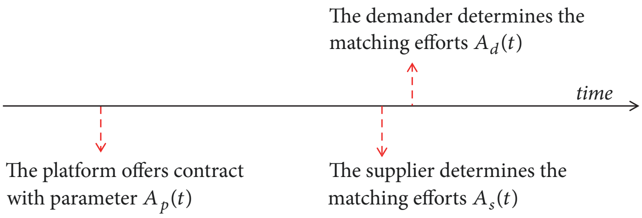

FIGURE 3: Sequence of events under the decentralised scenario.

functions. To obtain the optimal dynamic matching policy, we follow the literature [24, 25, 31] and use the Hamiltonian–Jacobi–Bellman (HJB) equations:

$$\rho V_s^N(R) = \max_{A_s \geq 0} \left\{ \pi_s D_s - C(A_s) + \omega R + V_s^{N\prime} \left(\alpha A_s + \beta A_p + \gamma A_d - \varphi R \right) \right\}, \tag{8}$$

$$\rho V_p^N(R) = \max_{A_p \geq 0, \varepsilon \geq 0} \left\{ cR - C(A_p) + V_p^{N\prime} \left(\alpha A_s + \beta A_p + \gamma A_d - \varphi R \right) \right\}, \tag{9}$$

$$\rho V_d^N(R) = \max_{A_d \geq 0} \left\{ \pi_d D_d - C(A_d) - (\omega + c) R + V_d^{N\prime} \left(\alpha A_s + \beta A_p + \gamma A_d - \varphi R \right) \right\}. \tag{10}$$

This puts us in a position to propose optimal strategies for the supply chain with decentralised decision-making. Proposition 1 characterises the equilibrium strategies.

Proposition 1. *In the decentralised system, the equilibrium results of the differential game between the supplier, platform, and demander are as follows.*

(i) The equilibrium matching efforts are given by

$$A_s^{N*} = \frac{\alpha (\pi_s \eta_s + \omega)}{\mu_s (\rho + \varphi)}, \tag{11}$$

$$A_d^{N*} = \frac{\gamma (\pi_d \eta_d - \omega - c)}{\mu_d (\rho + \varphi)}, \tag{12}$$

$$A_p^{N*} = \frac{\beta c}{\mu_p (\rho + \varphi)}. \tag{13}$$

(ii) The manufacturing resource-sharing level in the supply chain is given by

$$R^{N*} = K^N + \left(R_0 - K^N \right) e^{-\varphi t}, \tag{14}$$

where the parameter $K^N = \{\alpha^2 \mu_p \mu_d (\pi_s \eta_s + \omega) + \beta^2 \mu_s \mu_d c + \gamma^2 \mu_s \mu_p (\pi_d \eta_d - \omega - c)\} / (\mu_s \mu_p \mu_d \varphi (\varphi + \rho))$.

(iii) The optimal profit functions for the supplier, platform, and demander are given by

$$J_s^{N*} = e^{-\rho t} V_s^N \left(R^{N*} \right), \tag{15}$$

$$J_p^{N*} = e^{-\rho t} V_p^N \left(R^{N*} \right), \tag{16}$$

$$J_d^{N*} = e^{-\rho t} V_d^N \left(R^{N*} \right), \tag{17}$$

where the parameters a_1^N, a_2^N, a_3^N *and* b_1^N, b_2^N, b_3^N *are the coefficients of the linear value functions*

$$\begin{aligned} V_s^N \left(R^{N*} \right) &= a_1^Y R^{N*} + b_1^N \\ V_p^N \left(R^{Y*} \right) &= a_2^Y R^{N*} + b_2^N \\ V_d^N \left(R^{N*} \right) &= a_3^Y R^{N*} + b_3^N, \end{aligned} \tag{18}$$

which are determined in the proof for Proposition 1 (see the Appendix).

Proposition 1 shows that the sharing level R^{N*} is positive. This means that all party members are involved in the supply chain. The matching efforts A_s^{N*}, A_d^{N*}, and A_p^{N*} should be positive and decreasing at decay rate φ. In contrast, the matching efforts A_s^{N*}, A_d^{N*}, and A_p^{N*} increase in terms of effectiveness parameters α, β, and γ, respectively. This indicates that when the investment is efficient, the supplier, platform, and demander are motivated to invest more in supply-demand matching.

4. Optimal Strategies in the Centralised System

In this section, we examine the performance of a centralised supply chain. Supply chain members integrate to set the optimal matching efforts in view of maximising the total supply chain profit. In this game, $A_s(t)$, $A_d(t)$, and $A_p(t)$ are decision variables. We use the superscript 'I' to signify the centralised decision scenario.

Assuming an infinite time horizon and a positive discount rate ρ, the objective function of the supply chain in the centralised system is given as

$$\begin{aligned} J_{sc} = \max \quad & \int_0^\infty e^{-\rho t} \left\{ \pi_s D_s(t) + \pi_d D_d(t) - C(A_s(t)) - C\left(A_p(t)\right) - C(A_d(t)) \right\} dt \\ \text{s.t.} \quad & R(t)' = \alpha A_s(t) + \beta A_p(t) + \gamma A_d(t) - \varphi R(t), \\ & R(0) = R_0 . \end{aligned} \tag{19}$$

From this point forward, the time argument is omitted. Let ${V_{sc}}^{I}$ denote the supply chain system's value functions; the HJB equation is

$$\begin{aligned}\rho {V_{sc}}^{I}(R) = \max_{A_s,A_p,A_d} \Big\{ & \pi_s D_s + \pi_d D_d - C(A_s) \\ & - C(A_p) - C(A_d) \\ & + {V_{sc}}^{I\prime}(\alpha A_s + \beta A_p + \gamma A_d - \varphi x) \Big\}. \end{aligned} \tag{20}$$

We are now in a position to propose optimal strategies for the supply chain with centralised decision-making. Proposition 2 characterises the equilibrium strategies.

Proposition 2. *With centralised decision-making, the equilibrium results of the differential game between the supplier, the platform, and the demander are as follows.*

(i) The equilibrium matching efforts are given by

$${A_s}^{I*} = \frac{\alpha(\pi_s \eta_s + \pi_d \eta_d)}{\mu_s(\rho + \varphi)}, \tag{21}$$

$${A_d}^{I*} = \frac{\gamma(\pi_s \eta_s + \pi_d \eta_d)}{\mu_d(\rho + \varphi)}, \tag{22}$$

$${A_p}^{I*} = \frac{\beta(\pi_s \eta_s + \pi_d \eta_d)}{\mu_p(\rho + \varphi)}. \tag{23}$$

(ii) The sharing level of manufacturing resources in the supply chain is given by

$$R^{I*} = K^I + (R_0 - K^I) e^{-\varphi t}, \tag{24}$$

where the parameter $K^I = (\alpha^2 \mu_p \mu_d + \beta^2 \mu_s \mu_d + \gamma^2 \mu_s \mu_p)(\pi_s \eta_s + \pi_d \eta_d)/(\mu_s \mu_p \mu_d \varphi(\varphi + \rho))$.

(iii) The optimal profit function of the supply chain system is given by

$${J_{sc}}^{I*} = e^{-\rho t} {V_{sc}}^{I}(R^{I*}), \tag{25}$$

where the parameters a^I *and* b^I *are the coefficients of the linear function* ${V_{sc}}^{I}(R^{I*}) = a^I R^{N*} + b^I$*, which are determined in the Proof of Proposition 2 (see the Appendix).*

Proposition 3. *Compared with optimal strategies and profit functions in the decentralised and centralised systems, one has* ${A_s}^{I*} > {A_s}^{N*}$, ${A_d}^{I*} > {A_d}^{N*}$, ${A_p}^{I*} > {A_p}^{N*}$, *and* ${J_{sc}}^{I*} > {J_{sc}}^{N*}$.

We provide the proof for Proposition 3 in the Appendix. These relationships are derived through algebraic comparison. The matching efforts are higher in the centralised system, which means that the total profit is lower in the decentralised system. Hence, there is a need to design an appropriate contract to improve system efficiency.

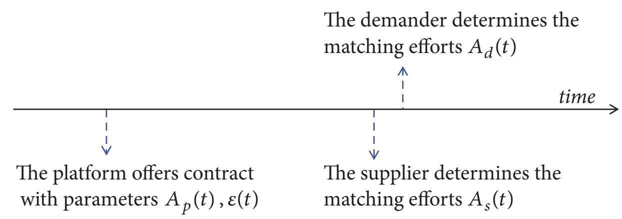

Figure 4: Sequence of the events under the coordination-contract scenario.

5. Optimal Strategies under the Coordination Contract

In this scenario, the platform is the channel leader and supports the demander's matching efforts. We use the superscript 'Y' to signify the coordination-contract scenario. $\varepsilon(t)$ denotes the platform's support rate, which represents the amount that the platform contributes to the demander's matching efforts within the interval $[0, 1]$. We are motivated by the coordination method used in [23] to develop a committed dynamic cost-sharing contract capable of coordinating the supply chain and improving the decentralised supply chain's performance. The contract provisions are structured as follows. In the game's first stage, the platform decides the matching efforts and the support rate $\varepsilon(t)$. In the second stage, both the supplier and demander make their decisions, respectively. In particular, the supplier determines the matching efforts $A_s(t)$ and the demander determines the matching efforts $A_d(t)$. The sequence of the events is shown in Figure 4.

Assuming an infinite time horizon and a positive discount rate ρ, the objective functionals of supply chain members under the coordination-contract scenario are

$$J_s^Y = \max_{A_s} \int_0^\infty e^{-\rho t} \{\pi_s D_s(t) - C(A_s(t)) + \omega R(t)\} \, dt, \tag{26}$$

$$\begin{aligned} J_p^Y = \max_{A_p,\varepsilon} \int_0^\infty e^{-\rho t} \Big\{ & cR(t) - C(A_p(t)) \\ & - \varepsilon(t) C(A_d(t)) \Big\} \, dt, \end{aligned} \tag{27}$$

$$\begin{aligned} J_d^Y = \max_{A_d} \int_0^\infty e^{-\rho t} \{ & \pi_d D_d(t) - (\omega + c) R \\ & - (1 - \varepsilon(t)) C(A_d) \} \, dt. \end{aligned} \tag{28}$$

From this point forward, the time argument is omitted. Let ${V_s}^{Y}$, ${V_p}^{Y}$, and ${V_d}^{Y}$ denote the players' value functions; the HJB equations are

$$\begin{aligned} \rho {V_s}^{Y}(R) = \max_{A_s \geq 0} \Big\{ & \pi_s D_s - C(A_s) + \omega R \\ & + {V_s}^{Y\prime}(\alpha A_s + \beta A_p + \gamma A_d - \varphi R) \Big\}, \end{aligned} \tag{29}$$

$$\begin{aligned} \rho {V_p}^{Y}(R) = \max_{A_p \geq 0, \varepsilon \geq 0} \Big\{ & cR - C(A_p) - \varepsilon C(A_d) \\ & + {V_p}^{Y\prime}(\alpha A_s + \beta A_p + \gamma A_d - \varphi R) \Big\}, \end{aligned} \tag{30}$$

$$\begin{aligned} \rho {V_d}^{Y}(R) = \max_{A_d \geq 0} \Big\{ & \pi_d D_d - (\omega + c) R - (1 - \varepsilon) C(A_d) \\ & + {V_d}^{Y\prime}(\alpha A_s + \beta A_p + \gamma A_d - \varphi R) \Big\}. \end{aligned} \tag{31}$$

We are now in a position to propose optimal strategies for the cost-sharing contract system. Proposition 4 characterises the equilibrium strategies.

Proposition 4. *Under the coordination-contract scenario, the equilibrium results of the differential game between the supplier, the platform, and the demander are as follows.*

(i) The equilibrium matching efforts and platform's support rate are given by

$$A_s^{Y*} = \frac{\alpha(\pi_s\eta_s + \omega)}{\mu_s(\rho+\varphi)}, \quad (32)$$

$$A_d^{Y*} = \frac{\gamma(\pi_d\eta_d - \omega + c)}{2\mu_d(\rho+\varphi)}, \quad (33)$$

$$A_p^{Y*} = \frac{\beta c}{\mu_p(\rho+\varphi)}, \qquad \varepsilon = \frac{-\pi_d\eta_d + \omega + 3c}{\pi_d\eta_d - \omega + c}. \quad (34)$$

(ii) The sharing level of manufacturing resources in the supply chain is given by

$$R^{Y*} = K^Y + \left(R_0 - K^Y\right)e^{-\varphi t}, \quad (35)$$

where the parameter $K^Y = (2\alpha^2\mu_p\mu_d(\pi_s\eta_s + \omega) + 2\beta^2 c\mu_s\mu_d + \gamma^2\mu_s\mu_p(\pi_d\eta_d - \omega + c))/2\mu_s\mu_p\mu_d\varphi(\varphi+\rho)$.

(iii) The optimal profit functions of supply chain members are given by

$$J_s^{Y*} = e^{-\rho t}V_s^Y\left(R^{Y*}\right), \quad (36)$$

$$J_p^{Y*} = e^{-\rho t}V_p^Y\left(R^{Y*}\right), \quad (37)$$

$$J_d^{Y*} = e^{-\rho t}V_d^Y\left(R^{Y*}\right), \quad (38)$$

where the parameters a_1^Y, a_2^Y, a_3^Y *and* b_1^Y, b_2^Y, b_3^Y *are the coefficients of the linear value functions*

$$V_s^Y\left(R^{Y*}\right) = a_1^Y R^{Y*} + b_1^Y$$
$$V_p^Y\left(R^{Y*}\right) = a_2^Y R^{Y*} + b_2^Y \quad (39)$$
$$V_d^Y\left(R^{Y*}\right) = a_3^Y R^{Y*} + b_3^Y,$$

which are determined in the proof of Proposition 4 (see the Appendix).

Next, we compare each supply chain member's profits and the total channel profits with the corresponding values in the above three scenarios. Our objective is to identify the effect of the cost-sharing contract on all channel members' profits to determine whether the cost-sharing contract increases profits and thus improves coordination. For notational convenience, let $K_1 = \max\{0, (\pi_d\eta_d - \omega)/3\}$ and $K_2 = \pi_d\eta_d - \omega$; the interval (K_1, K_2) is the coordination contract's feasible region. We then arrive at the following proposition.

Proposition 5. *The strategies and payoffs in the decentralised scenario (N), cost-sharing contract scenario (Y), and centralised decision scenario (I) are related as follows:*

(i) The supplier equilibrium matching efforts, $A_s^{N*} = A_s^{Y*} < A_s^{I*}$.

(ii) The platform equilibrium matching efforts, $A_p^{N*} = A_p^{Y*} < A_p^{I*}$.

(iii) The demander equilibrium matching efforts, $A_d^{N*} < A_d^{Y*} < A_d^{I*}$.

(iv) The optimal profits, $J_s^{N*} < J_s^{Y*}$, $J_p^{N*} < J_p^{Y*}$, $J_d^{N*} < J_d^{Y*}$, *and* $J_{sc}^{N*} < J_{sc}^{Y*}$ *for* $K_1 < c < K_2$.

We provide the proof for Proposition 5 in the Appendix. These relationships are derived through algebraic comparison. Proposition 5 shows that all supply chain members incur higher profits in the cost-sharing contract scenario than the decentralised decision-making scenario. Clearly, cost-sharing with the platform provides the greatest benefit to the demander: when the platform manager covers any share of the matching costs, it helps improve the demander's profitability. As the matching costs are lowered, the demander can offer a higher level of matching effort, which subsequently drives up market demand for the resource or service. This increase in market demand more than compensates for the cost shared by the platform.

This result illustrates why matching involves increased collaboration between the demander and the platform manager through cost-sharing contracts and other mechanisms. However, because the comparison of the supplier, platform, demander, and supply chain profits poses some degree of analytical complexity, we now turn to numerical computation to verify our theoretical findings.

6. Numerical Example

In this section, we conduct numerical analyses to gain managerial insights. Set $\pi_s = 5$, $\pi_d = 5$, $\mu_s = 10$, $\mu_p = 15$, $\mu_d = 14$, $\eta_s = 0.9$, $\eta_d = 1.7$, $x_0 = 0.25$, $\alpha = 2$, $\beta = 2$, $\gamma = 3$, $\varphi = 0.5$, $\theta = 0.5$, $a = 5$, $c = 4$, $\omega = 0.6$, and $\rho = 0.9$. In Section 6.1, we compare the operational performance of the decentralised (N), cost-sharing contract (Y), and centralised decision (I) scenarios, focusing on the dynamic strategies, the sharing level and profits. In Section 6.2, we examine the impacts of the platform transaction fee and purchasing cost on the feasible region of the corresponding contract and obtain some useful insights.

Before we proceed, recall that the profit functions are linear in the value function V and can be written as an exponential function multiplied by the value function, i.e., $J = e^{-\rho t}V$, which makes it sufficient for comparison. Thus, to compare J_s^*, J_p^*, J_d^*, and J_{sc}^*, we compare the values of V_s^*, V_p^*, V_d^*, and V_{sc}^*, respectively. Define $\Delta V_s = V_s^{Y*} - V_s^{N*}$, $\Delta V_p = V_p^{Y*} - V_p^{N*}$, $\Delta V_d = V_d^{Y*} - V_d^{N*}$ and $\Delta J_s = J_s^{Y*} - J_s^{N*}$, $\Delta J_p = J_p^{Y*} - J_p^{N*}$, $\Delta J_d = J_d^{Y*} - J_d^{N*}$. Similarly, a comparison between ΔV_s, ΔV_p, and ΔV_d is equivalent to a comparison between the profit functions ΔJ_s, ΔJ_p, and ΔJ_d, respectively.

Table 2: Optimal strategies in supply chain systems.

	Decentralised (N)	Cost sharing (Y)	Centralised (I)
A_s^*	0.73	0.73	1.86
A_p^*	0.41	0.41	1.99
A_d^*	0.56	0.85	1.24

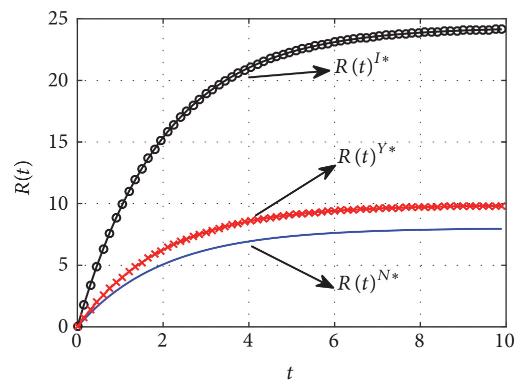

Figure 5: Dynamic change in manufacturing resources sharing level.

6.1. Optimal Solutions

6.1.1. Comparisons of Optimal Solutions. According to Propositions 1, 2, and 4, we can obtain the optimal matching efforts and sharing level in the decentralised, cost-sharing contract, and centralised decision scenarios. From Table 2, we can see that ${A_s}^{N*} = {A_s}^{Y*} < {A_s}^{I*}$, ${A_p}^{N*} = {A_p}^{Y*} < {A_p}^{I*}$, and ${A_d}^{N*} < {A_d}^{Y*} < {A_d}^{I*}$. The matching efforts in the centralised structure are higher than those in the decentralised scenario, and the optimal matching efforts in the decentralised scenario are equal to or less than those in the cost-sharing contract scenario. This is consistent with the conclusions in Proposition 5. Figure 5 shows changes to the sharing level over time. Here, the corresponding optimal resources sharing level are given as follows:

$$R(t)^* = \begin{cases} 8.02 - 5.52e^{-0.5t}, & \text{Decentralized } (N) \\ 9.9 - 7.4e^{-0.5t}, & \text{Cost sharing } (Y) \\ 24.32 - 21.82e^{-0.5t}, & \text{Centralized } (I). \end{cases} \quad (40)$$

Figure 5 shows that the optimal resource-sharing levels in the centralised decision-making system are higher than those in the decentralised and cost-sharing scenarios, as the matching efforts are higher in the centralised decision system.

6.1.2. Comparison of Profits. In this subsection, we compare the profits across the three models; we provide the results in Figure 6. The profit in the centralised decision scenario is the highest, followed by the cost-sharing contract and the decentralised scenario, respectively, which verifies Proposition 5.

Figure 7 presents a comparison between each supply chain member's profits before and after cost-sharing. The equilibrium values in the cost-sharing contract are in the

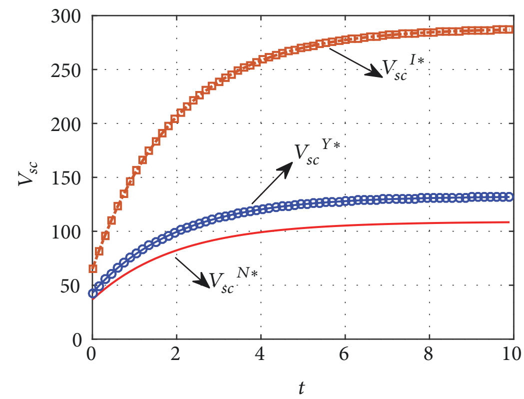

Figure 6: Optimal profit comparison in scenarios N, Y, and I.

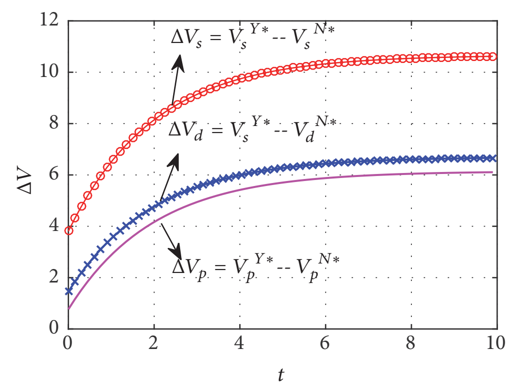

Figure 7: Profit comparison for supply chain parties before and after cost-sharing.

following order in comparison with the decentralised supply chain values: ${V_s}^{Y*} > {V_s}^{N*}$, ${V_p}^{Y*} > {V_p}^{N*}$, ${V_d}^{Y*} > {V_d}^{N*}$ for $K_1 < c < K_2$. This indicates that the supplier, platform manager, and demander all enjoy higher profits in the cost-sharing contract than in the decentralised supply chain case. The cost-sharing contract effectively improves the performance of the decentralised supply chain. The cost-sharing contract achieves Pareto improvement for the supplier, the platform manager, and the demander under certain conditions. Any share of matching costs helps improve the demander's profitability. As such, the demander can provide a higher matching effort; this increases market demand, which more than compensates for the cost shared by the platform.

Moreover, the supplier's profit increases are the highest, followed by the demander and the platform manager, respectively. In the cost-sharing contract scenario, the platform supports the demander's matching efforts and the supplier does not incur any additional matching costs. Cost-sharing lowers the demander's burden in the supply chain structure; the demander thus enjoys greater benefit from the matching decision (see Columns 1 and 2 in Table 2). A comparison of the platform's profit shows that the platform manager incurs higher profits in the cost-sharing contract than the decentralised scenario. The contract thus also benefits the platform. These results illustrate why matching involves

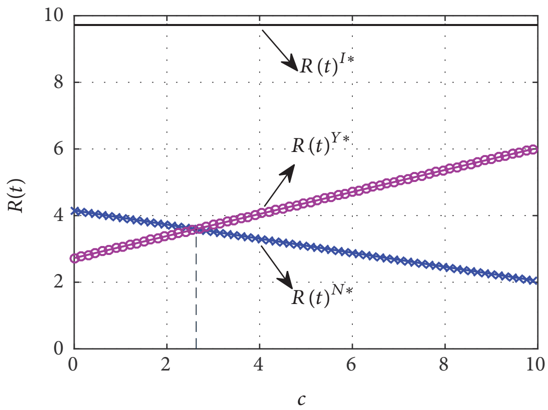

Figure 8: Impact of the transaction fee on the level of manufacturing resource sharing.

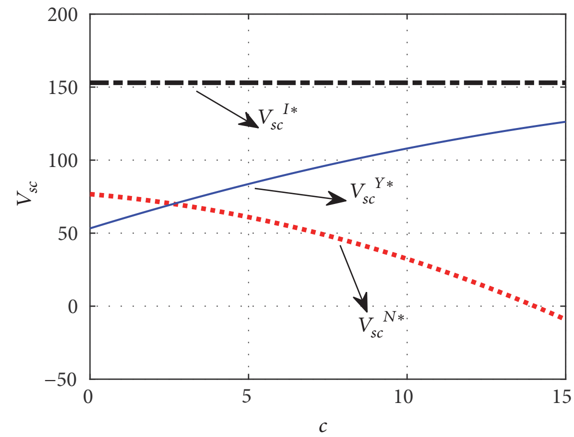

Figure 9: Impact of the transaction fee on optimal profit.

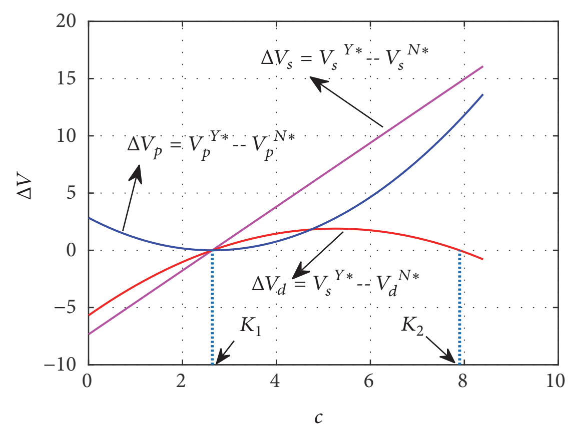

Figure 10: Pareto improvement effect of the transaction fee for the cost-sharing contract.

increased collaboration between the demander and the platform through cost-sharing and other mechanisms.

6.2. Sensitivity Analysis of the Platform Transaction Fee (c). We first investigate the effects of platform transaction fees on the sharing level. In Figure 8, the sharing level $R(t)^*$ is plotted as a function of the platform transaction fee c. The sharing level decreases as the transaction fee increases in the decentralised decision system due to the fact that the demander's marginal profit decreases with the increase of the transaction fee. As in Section 2.1, the demand function depends on the marginal profit and the sharing level. Thus, the larger the transaction fee, the smaller the market demand. Accordingly, in Figure 9, we see that the profit in the decentralised decision system decreases when the transaction fee c is raised. In contrast, the sharing level increases in tandem with the platform transaction fee in the cost-sharing contract scenario because the platform's support rate $\varepsilon^*(t)$ increases with the transaction fee c (see Proposition 4). The larger the transaction fee, the larger the support rate. As such, the demander has a greater incentive to increase its matching efforts; this drives up the market demand, which more than compensates for the cost shared by the platform. Accordingly, Figure 9 shows that in the cost-sharing contract scenario, profit increases with c.

However, we also find that the cost-sharing contract does not always achieve Pareto improvement for all parties (i.e., the value can fall outside the feasible region). Figure 10 shows that only when the value of c is between K_1 and K_2 can the cost-sharing contract adequately coordinate the supply chain such that all parties benefit. Specifically, when the purchasing cost ω increases, the win-win region becomes smaller in Figure 11. This implies that, as the value of ω increases, the degree of flexibility in coordinating the supply chain decreases.

7. Conclusions

In this paper, we discussed the challenges of supply-demand matching for manufacturing resource- and service-sharing by considering the sharing level in a complex and dynamic environment. Applying optimal control theory, we identified the optimal matching strategies for decentralised, centralised and cost-sharing contract systems. Our main contribution lies in the following. First, we considered the dynamic evolution feature of the sharing level, which we set as a state variable. Second, we optimised matching-effort strategies through differential game models to coordinate the decentralised supply chain. Finally, we conducted a numerical analysis to illustrate the effect of the platform transaction fee and purchasing costs on equilibria and coordination.

In particular, we obtained the following results. (1) A cost-sharing contract effectively improves the performance of the decentralised supply chain. All channel members (i.e., the manufacturing resource or service supplier, platform manager, and resource or service demander) incur higher profits in the cost-sharing contract system than the decentralised system. (2) The cost-sharing contract does not always achieve Pareto improvement for all parties. (3) Numerical analysis shows that the platform transaction fee and purchasing costs affect the win-win region and optimal strategies. A larger purchasing cost will limit the degree of flexibility with which supply chain members coordinate the supply chain, thus providing manufacturers and the service platform with guidance to improve profitability. Our study contributes to

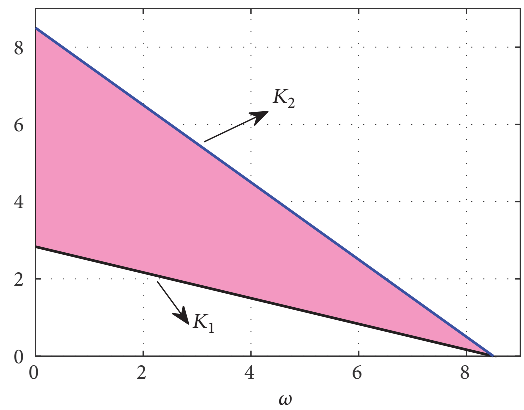

FIGURE 11: Impact of ω on the feasible region of the cost-sharing contract.

the burgeoning field of idle manufacturing resource sharing within supply chains and collaboration between channel partners.

Appendix

A.

Proof of Proposition 1. We need to establish the existence of bounded and continuously differentiable value functions ${V_s}^N$, ${V_p}^N$, and ${V_d}^N$ such that it is a unique solution R^N to differential equation (1) and the HJB equations. We first determine the players' necessary conditions from the HJBs. Because the game is played à la Stackelberg and the platform is the leader, we first derive the decision variables for the supplier and the demander for the second game stage. The optimisation problem of supplier is given as

$$\begin{aligned} J_s^N = \max_{A_s} \quad & \int_0^\infty e^{-\rho t} \left\{ \pi_s D_s(t) - C(A_s(t)) + \omega R(t) \right\} dt \\ s.t. \quad & R(t)' = \alpha A_s(t) + \beta A_p(t) + \gamma A_d(t) - \varphi R(t), \\ & R(0) = R_0 . \end{aligned} \tag{A.1}$$

Let the value functions ${V_s}^N = \max_{A_s} \int_t^\infty e^{-\rho(\tau - t)} \{\pi_s D_s(t) - C(A_s(t)) + \omega R(t)\} d\tau$; the optimal profit function of manufacturing resources supplier then is given as

$$J_s^N = e^{-\rho t} {V_s}^N. \tag{A.2}$$

Similarly, the optimal profit function of the platform and demander are given by

$$\begin{aligned} J_p^N &= e^{-\rho t} {V_p}^N, \\ J_d^N &= e^{-\rho t} {V_d}^N. \end{aligned} \tag{A.3}$$

The supplier's HJB is

$$\rho {V_s}^N(R) = \max_{A_s \ge 0} \left\{ \pi_s D_s - \frac{\mu_s {A_s}^2}{2} + \omega R + {V_s}^{N'} R' \right\}, \tag{A.4}$$

and its maximisation provides the necessary condition for matching efforts:

$${A_s}^N = \frac{\alpha {V_s}^{N'}}{\mu_s}. \tag{A.5}$$

Similarly, the demander's HJB is

$$\begin{aligned} & \rho {V_d}^N(R) \\ & = \max_{A_d \ge 0} \left\{ \pi_d D_d - \frac{\mu_d {A_d}^2}{2} - (\omega + c) R + {V_d}^{N'} R' \right\}, \end{aligned} \tag{A.6}$$

and its maximisation provides the necessary condition for matching efforts:

$$A_d^N = \frac{\gamma {V_d}^{N'}}{\mu_d}. \tag{A.7}$$

Substituting (A.5) and (A.7) into the platform's HJB gives

$$\rho {V_p}^N(R) = \max_{F \ge 0} \left\{ cR - \frac{\mu_p A_p^2}{2} + {V_p}^{N'} R' \right\}, \tag{A.8}$$

and by performing the maximisation of the right-hand side we obtain

$$A_p^N = \frac{\beta {V_p}^{N'}}{\mu_p}. \tag{A.9}$$

By inserting (A.5), (A.7), and (A.9) inside the HJBs, we obtain the following three algebraic equations:

$$\begin{aligned} \rho {V_s}^N(R) = {} & \pi_s (a - \pi_s + \theta \pi_d - \theta \pi_s + \eta_s R) \\ & + \frac{\left(\alpha {V_s}^{N'}\right)^2}{(2\mu_s)} + \omega R \\ & + {V_s}^{N'} \left(\frac{\beta^2 {V_p}^{N'}}{\mu_p} + \frac{\gamma^2 {V_d}^{N'}}{\mu_d} - \varphi R \right), \end{aligned} \tag{A.10}$$

$$\begin{aligned} \rho {V_p}^N(R) = {} & cR + \frac{\left(\beta {V_p}^{N'}\right)^2}{2\mu_p} \\ & + {V_p}^{N'} \left(\frac{\alpha^2 {V_s}^{N'}}{\mu_s} + \frac{\gamma^2 {V_d}^{N'}}{\mu_d} - \varphi R \right), \end{aligned} \tag{A.11}$$

$$\begin{aligned} \rho {V_d}^N(R) = {} & \pi_d (a - \pi_d + \theta \pi_s - \theta \pi_d + \pi_d R) \\ & + \frac{\left(\gamma {V_d}^{N'}\right)^2}{(2\mu_d)} - (\omega + c) R \\ & + {V_d}^{N'} \left(\frac{\alpha^2 {V_s}^{N'}}{\mu_s} + \frac{\beta^2 {V_p}^{N'}}{\mu_p} - \varphi R \right). \end{aligned} \tag{A.12}$$

Following the literature [24, 25, 31], we obtain the following linear forms for the value functions:

$$\begin{aligned} V_s^N(R) &= a_1^N R + b_1^N, \\ V_p^N(R) &= a_2^N R + b_2^N, \\ V_d^N(R) &= a_3^N R + b_3^N, \end{aligned} \quad \text{(A.13)}$$

in which a_1^N, a_2^N, and a_3^N and b_1^N, b_2^N, and b_3^N are constants. From formula (A.13), we have

$$\begin{aligned} V_s^{N\prime} &= a_1^N, \\ V_p^{N\prime} &= a_2^N, \\ V_d^{N\prime} &= a_3^N. \end{aligned} \quad \text{(A.14)}$$

We substitute V_s^N, V_p^N, and V_d^N from (A.13) and their derivatives from (A.14) into ((A.10)-(A.12)) and collect terms corresponding to R. By solving the algebraic equations, we have

$$\begin{aligned} a_1^N &= \frac{(\pi_s\eta_s + \omega)}{(\rho + \varphi)}, \\ a_2^N &= \frac{c}{(\rho + \varphi)}, \\ a_3^N &= \frac{(\pi_d\eta_d - \omega - c)}{(\rho + \varphi)} \\ b_1^N &= \frac{\pi_s(a - \pi_s + \theta\pi_d - \theta\pi_s)}{\rho} \\ &\quad + \frac{\alpha^2}{2\mu_s\rho}\left(\frac{\pi_s\eta_s + \omega}{\rho + \varphi}\right)^2 + \frac{\pi_s\eta_s + \omega}{\rho(\rho + \varphi)^2}\left[\frac{\beta^2 c}{\mu_p}\right. \\ &\quad \left. + \frac{\gamma^2(\pi_d\eta_d - \omega - c)}{\mu_d}\right] \\ b_2^N &= \frac{\beta^2}{2\mu_p\rho}\left(\frac{c}{\rho + \varphi}\right)^2 \\ &\quad + \frac{c}{\rho(\rho + \varphi)^2}\left[\frac{\alpha^2(\pi_s\eta_s + \omega)}{\mu_s}\right. \\ &\quad \left. + \frac{\gamma^2(\pi_d\eta_d - \omega - c)}{\mu_d}\right] \\ b_3^N &= \frac{\pi_d(a - \pi_d + \theta\pi_s - \theta\pi_d)}{\rho} \\ &\quad + \frac{\gamma^2}{2\mu_d\rho}\left(\frac{\pi_d\eta_d - \omega - c}{\rho + \varphi}\right)^2 \\ &\quad + \frac{\pi_d\eta_d - \omega - c}{\rho(\rho + \varphi)^2}\left[\frac{\alpha^2(\pi_s\eta_s + \omega)}{\mu_s} + \frac{\beta^2 c}{\mu_p}\right]. \end{aligned} \quad \text{(A.15)}$$

Substituting (A.14) into (A.5), (A.7), and (A.9), the equilibrium matching efforts are given by

$$\begin{aligned} A_s^{N*} &= \frac{\alpha(\pi_s\eta_s + \omega)}{\mu_s(\rho + \varphi)}, \\ A_p^{N*} &= \frac{\beta c}{\mu_p(\rho + \varphi)}, \\ A_d^{N*} &= \frac{\gamma(\pi_d\eta_d - \omega - c)}{\mu_d(\rho + \varphi)}. \end{aligned} \quad \text{(A.16)}$$

Next, substituting A_s^{N*}, A_p^{N*}, and A_d^{N*} into differential equation (1) and using the initial conditions of (1), the general solution of the differential equation for the sharing level R is

$$R^{N*} = K^N + \left(R_0 - K^N\right)e^{-\varphi t}, \quad \text{(A.17)}$$

where $K^N = \{\alpha^2\mu_p\mu_d(\pi_s\eta_s + \omega) + \beta^2\mu_s\mu_d c + \gamma^2\mu_s\mu_p(\pi_d\eta_d - \omega - c)\}/(\mu_s\mu_p\mu_d\varphi(\varphi + \rho))$.

This completes the proof. □

B.

Proof of Proposition 2. We need to establish the existence of bounded and continuously differentiable value function V_{sc}^I such that there exists a unique solution R^I to differential equation (1) and the HJB equations. We first determine the players' necessary conditions from the HJBs. The optimisation problem of supplier is given as

$$\begin{aligned} J_{sc}^I = \max_{A_s, A_p, A_d} \quad & \int_0^\infty e^{-\rho t}\left\{\pi_s D_s + \pi_d D_d - C(A_s) - C(A_p) - C(A_d)\right\} dt \\ s.t. \quad & R(t)' = \alpha A_s(t) + \beta A_p(t) + \gamma A_d(t) - \varphi R(t), \\ & R(0) = R_0. \end{aligned} \quad \text{(B.1)}$$

Let the value functions ${V_{sc}}^{I} = \max_{A_s,A_p,A_d} \int_t^{\infty} e^{-\rho(\tau-t)}\{\pi_s D_s + \pi_d D_d - C(A_s) - C(A_p) - C(A_d)\} d\tau$; the optimal profit function of supply chain then is given as

$$J_{sc}^{I} = e^{-\rho t} {V_{sc}}^{I}. \tag{B.2}$$

The supply chain's HJB is

$$\begin{aligned} \rho {V_{sc}}^{I}(R) = \max_{A_s,A_p,A_d} \Big\{ & \pi_s D_s + \pi_d D_d - C(A_s) \\ & - C(A_p) - C(A_d) + {V_{sc}}^{I\prime} R' \Big\}, \end{aligned} \tag{B.3}$$

and its maximisation provides the necessary condition for matching efforts:

$$\begin{aligned} A_s^I &= \frac{\alpha {V_{sc}}^{I\prime}}{\mu_s}, \\ A_p^I &= \frac{\beta {V_{sc}}^{I\prime}}{\mu_p}, \\ A_d^I &= \frac{\gamma {V_{sc}}^{I\prime}}{\mu_d}. \end{aligned} \tag{B.4}$$

By inserting (B.4) inside the HJB we obtain the following algebraic equations:

$$\begin{aligned} \rho {V_{sc}}^{I}(R) = \max_{A_s,A_p,A_d} \Bigg\{ & \pi_s (a - \pi_s + \theta\pi_d - \theta\pi_s + \eta_s R) \\ & + \pi_d (a - \pi_d + \theta\pi_s - \theta\pi_d + \eta_d R) + \frac{(\alpha {V_{sc}}^{I\prime})^2}{(2\mu_s)} \\ & + \frac{(\beta {V_{sc}}^{I\prime})^2}{(2\mu_p)} + \frac{(\gamma {V_{sc}}^{I\prime})^2}{(2\mu_d)} + {V_{sc}}^{I\prime} \varphi R \Bigg\}. \end{aligned} \tag{B.5}$$

Thus, we obtain the following linear forms for the value functions:

$${V_{sc}}^{I}(R) = a^I R + b^I, \tag{B.6}$$

in which a^I and b^I are constants. From formula (B.6), we have

$${V_{sc}}^{I\prime} = a^I. \tag{B.7}$$

We substitute ${V_{sc}}^{I}$ from (B.6), as well as its derivative from (B.7), into (B.5), and collect terms corresponding to R. By solving the algebraic equations, we have

$$\begin{aligned} a^I &= \frac{(\pi_s\eta_s + \pi_d\eta_d)}{(\rho + \varphi)} \\ b^I &= \frac{\pi_s (a - \pi_s + \theta\pi_d - \theta\pi_s)}{\rho} \\ &\quad + \frac{\pi_d (a - \pi_d + \theta\pi_s - \theta\pi_d)}{\rho} \\ &\quad + \frac{1}{2\rho} \left(\frac{\pi_s\eta_s + \pi_d\eta_d}{\rho + \varphi} \right)^2 \left(\frac{\alpha^2}{\mu_s} + \frac{\beta^2}{\mu_p} + \frac{\gamma^2}{\mu_d} \right) \end{aligned} \tag{B.8}$$

Substituting ((B.7)-(B.8)) into (B.4) the equilibrium matching efforts are given by

$$\begin{aligned} {A_s}^{I*} &= \frac{\alpha (\pi_s\eta_s + \pi_d\eta_d)}{\mu_s (\rho + \varphi)}, \\ {A_p}^{I*} &= \frac{\beta (\pi_s\eta_s + \pi_d\eta_d)}{\mu_p (\rho + \varphi)}, \\ {A_d}^{I*} &= \frac{\gamma (\pi_s\eta_s + \pi_d\eta_d)}{\mu_d (\rho + \varphi)}. \end{aligned} \tag{B.9}$$

Next, substituting ${A_s}^{I*}, {A_p}^{I*}, {A_d}^{I*}$ into differential equation (1) and using the initial conditions of (1), the general solution of the differential equation for the sharing level R is

$$R^{I*} = K^I + (R_0 - K^I) e^{-\varphi t}, \tag{B.10}$$

where $K^I = (\alpha^2\mu_p\mu_d + \beta^2\mu_s\mu_d + \gamma^2\mu_s\mu_p)(\pi_s\eta_s + \pi_d\eta_d)/(\mu_s\mu_p\mu_d\varphi(\varphi + \rho))$. This completes the proof. □

C.

Proof of Proposition 3. To prove the first item, we need to establish that all decision variables of Proposition 1 are positive, i.e., ${A_d}^{N*} > 0$. This implies, in turn, $\pi_d\eta_d - \omega - c > 0$ (from (A.16)). Straightforward comparisons, using the values in (A.16) and (B.9), lead to the results:

$${A_s}^{I*} - {A_s}^{N*} = \frac{\alpha (\pi_d\eta_d - \omega)}{(\mu_s (\rho + \varphi))} > 0, \tag{C.1}$$

$${A_p}^{I*} - {A_p}^{N*} = \frac{\beta (\pi_s\eta_s + \pi_d\eta_d - c)}{(\mu_p (\rho + \varphi))} > 0, \tag{C.2}$$

$${A_d}^{I*} - {A_d}^{N*} = \frac{\gamma (\pi_s\eta_s + \omega + c)}{(\mu_d (\rho + \varphi))} > 0, \tag{C.3}$$

$$\begin{aligned} {J_{sc}}^{I*} - {J_{sc}}^{N*} &= e^{-\rho t} \left({V_{sc}}^{I*} - {V_{sc}}^{N*} \right) \\ &= \frac{3\gamma^2 (\pi_s\eta_s + \pi_d\eta_d)(\pi_s\eta_s + \omega - c) + \gamma^2 (\pi_d\eta_d - \omega + c)^2}{8\mu_d\rho (\rho + \varphi)^2} \\ &> 0. \end{aligned} \tag{C.4}$$

This completes the proof. □

D.

Proof of Proposition 4. We need to establish the existence of bounded and continuously differentiable value functions ${V_s}^Y$, ${V_p}^Y$, and ${V_d}^Y$ such that it is a unique solution R^Y to differential equation (1) and the HJB equations. We first determine the players' necessary conditions from the HJBs. Because the game is played à la Stackelberg and the platform is the leader, we first derive the decision variables for the supplier and the demander for the second game stage. The optimisation problem of supplier is given as

$$\begin{aligned} J_s^Y = \max_{A_s} \quad & \int_0^\infty e^{-\rho t} \left\{\pi_s D_s(t) - C\left(A_s(t)\right) + \omega R(t)\right\} dt \\ s.t. \quad & R(t)' = \alpha A_s(t) + \beta A_p(t) + \gamma A_d(t) - \varphi R(t), \\ & R(0) = R_0 . \end{aligned} \tag{D.1}$$

Let the value functions ${V_s}^Y = \max_{A_s} \int_t^\infty e^{-\rho(\tau - t)}\{\pi_s D_s(t) - C(A_s(t)) + \omega R(t)\} d\tau$; the optimal profit function of manufacturing resources supplier then is given as

$$J_s^Y = e^{-\rho t} {V_s}^Y. \tag{D.2}$$

Similarly, the optimal profit function of the platform and demander are given by

$$\begin{aligned} J_p^Y &= e^{-\rho t} {V_p}^Y, \\ {J_d}^Y &= e^{-\rho t} {V_d}^Y. \end{aligned} \tag{D.3}$$

The supplier's HJB is

$$\rho {V_s}^Y(R) = \max_{A_s \geq 0} \left\{\pi_s D_s - \frac{\mu_s {A_s}^2}{2} + \omega R + {V_s}^{N'} R'\right\}, \tag{D.4}$$

and its maximisation provides the necessary condition for matching efforts:

$$A_s^Y = \frac{\alpha {V_s}^{Y'}}{\mu_s}. \tag{D.5}$$

Similarly, the demander's HJB is

$$\begin{aligned} \rho {V_d}^Y(R) = \max_{A_d \geq 0} & \left\{\pi_d D_d - \frac{(1-\varepsilon)\mu_d {A_d}^2}{2} \right. \\ & \left. - (\omega + c) R + {V_d}^{N'} R'\right\}, \end{aligned} \tag{D.6}$$

and its maximisation provides the necessary condition for matching efforts:

$$A_d^Y = \frac{\gamma {V_d}^{Y'}}{(\mu_d (1-\varepsilon))}. \tag{D.7}$$

Substituting (D.5); (D.7) into the platform's HJB gives

$$\begin{aligned} \rho {V_p}^Y(R) = \max_{A_p \geq 0, \varepsilon \geq 0} & \left\{cR - \frac{\mu_p {A_p}^2}{2} - \frac{\varepsilon\left(\gamma {V_d}^{Y'}\right)^2}{2\mu_d (1-\varepsilon)^2} \right. \\ & \left. + {V_p}^{Y'}\left(\frac{\alpha^2 {V_s}^{Y'}}{\mu_s} + \beta A_p + \frac{\gamma^2 {V_d}^{Y'}}{\mu_d (1-\varepsilon)} - \varphi R\right)\right\}, \end{aligned} \tag{D.8}$$

while performing the maximisation of the right-hand side we obtain

$$\begin{aligned} A_p^Y &= \frac{\beta {V_p}^{Y'}}{\mu_p}, \\ \varepsilon &= \frac{\left(2{V_p}^{Y'} - {V_d}^{Y'}\right)}{\left(2{V_p}^{Y'} + {V_d}^{Y'}\right)}. \end{aligned} \tag{D.9}$$

By inserting (D.5), (D.7), and (D.9) inside the HJBs, we obtain the following three algebraic equations:

$$\begin{aligned} & \rho {V_s}^Y(R) \\ & = \pi_s (a - \pi_s + \theta\pi_d - \theta\pi_s + \eta_s R) + \frac{\left(\alpha {V_s}^{Y'}\right)^2}{(2\mu_s)} \\ & \quad + \omega R \\ & \quad + {V_s}^{Y'}\left(\frac{\beta^2 {V_p}^{Y'}}{\mu_p} + \frac{\gamma^2\left(2{V_p}^{Y'} + {V_d}^{Y'}\right)}{(2\mu_d)} - \varphi R\right), \end{aligned} \tag{D.10}$$

$$\begin{aligned} & \rho {V_p}^Y(R) \\ & = cR + \frac{\left(\beta {V_p}^{Y'}\right)^2}{\left(2\mu_p\right)} + \frac{\gamma^2\left(2{V_p}^{Y'} + {V_d}^{Y'}\right)^2}{(8\mu_d)} \\ & \quad + {V_p}^{Y'}\left(\frac{\alpha^2 {V_s}^{Y'}}{\mu_s} - \varphi R\right), \end{aligned} \tag{D.11}$$

$$\begin{aligned} & \rho {V_d}^N(R) \\ & = p_d (a - p_d + \theta p_s - \theta p_d + \eta_d R) \\ & \quad + \frac{\gamma^2 {V_d}^{Y'}\left(2{V_p}^{Y'} + {V_d}^{Y'}\right)}{(4\mu_d)} - (\omega + c) R \\ & \quad + {V_d}^{Y'}\left(\frac{\alpha^2 {V_s}^{Y'}}{\mu_s} + \frac{\beta^2 {V_p}^{Y'}}{\mu_p} - \varphi R\right), \end{aligned} \tag{D.12}$$

Thus, we obtain the following linear forms for the value functions:

$$V_s^Y(R) = a_1^Y R + b_1^Y,$$
$$V_p^Y(R) = a_2^Y R + b_2^Y, \quad \text{(D.13)}$$
$$V_d^Y(R) = a_3^Y R + b_3^Y,$$

in which a_1^Y, a_2^Y, and a_3^Y and b_1^Y, b_2^Y, and b_3^Y are constants. From formula (D.13), we have

$$V_s^{Y\prime} = a_1^Y,$$
$$V_p^{Y\prime} = a_2^Y, \quad \text{(D.14)}$$
$$V_d^{Y\prime} = a_3^Y.$$

We substitute V_s^Y, V_p^Y, V_d^Y from (D.13), as well as their derivatives from (D.14), into ((D.10)-(D.12)), and collect terms corresponding to R. By solving the algebraic equations, we have

$$\begin{aligned}
a_1^Y &= \frac{(\pi_s\eta_s + \omega)}{(\rho+\varphi)},\\
a_2^Y &= \frac{c}{(\rho+\varphi)},\\
a_3^Y &= \frac{(\pi_d\eta_d - \omega - c)}{\rho} + \varphi\\
b_1^Y &= \frac{\pi_s(a - \pi_s + \theta\pi_d - \theta\pi_s)}{\rho} + \frac{\alpha^2(\pi_s\eta_s+\omega)^2}{2\mu_s\rho(\rho+\varphi)^2}\\
&\quad + \frac{\pi_s\eta_s+\omega}{\rho(\rho+\varphi)^2}\left[\frac{\beta^2 c}{\mu_p} + \frac{\gamma^2(\pi_d\eta_d - \omega + c)}{2\mu_d}\right]\\
b_2^Y &= \frac{\beta^2}{2\mu_p\rho}\left(\frac{c}{\rho+\varphi}\right)^2 + \frac{\gamma^2(\pi_d\eta_d-\omega+c)^2}{8\mu_d\rho(\rho+\varphi)^2}\\
&\quad + \frac{c\alpha^2(\pi_s\eta_s+\omega)}{\mu_s\rho(\rho+\varphi)^2}\\
b_3^Y &= \frac{\pi_d(a - \pi_d + \theta\pi_s - \theta\pi_d)}{\rho}\\
&\quad + \frac{\gamma^2(\pi_d\eta_d-\omega-c)(\pi_d\eta_d-\omega+c)}{4\mu_d\rho(\rho+\varphi)^2}\\
&\quad + \frac{\pi_d\eta_d-\omega-c}{\rho(\rho+\varphi)^2}\left[\frac{\alpha^2(\pi_s\eta_s+\omega)}{\mu_s} + \frac{\beta^2 c}{\mu_p}\right]
\end{aligned} \quad \text{(D.15)}$$

Substituting (D.14) into (D.5), (D.7), and (D.9), the equilibrium matching efforts are given by

$$\begin{aligned}
A_s^{Y*} &= \frac{\alpha(\pi_s\eta_s+\omega)}{\mu_s(\rho+\varphi)},\\
A_p^{Y*} &= \frac{\beta c}{\mu_p(\rho+\varphi)},\\
\varepsilon &= \frac{3c - \pi_d\eta_d + \omega}{\pi_d\eta_d - \omega + c},\\
A_d^{Y*} &= \frac{\gamma(\pi_d\eta_d - \omega + c)}{2\mu_d(\rho+\varphi)}.
\end{aligned} \quad \text{(D.16)}$$

Next, substituting A_s^{Y*}, A_p^{Y*}, and A_d^{Y*} into differential equation (1) and using the initial conditions of (1), the general solution of the differential equation for the sharing level R is

$$R^{Y*} = K^Y + \left(R_0 - K^Y\right)e^{-\varphi t}, \quad \text{(D.17)}$$

where $K^Y = \{2\alpha^2\mu_p\mu_d(\pi_s\eta_s+\omega) + 2\beta^2 c\mu_s\mu_d + \gamma^2\mu_s\mu_p(\pi_d\eta_d - \omega + c)\}/(2\mu_s\mu_p\mu_d\varphi(\varphi+\rho))$.

This completes the proof. □

E.

Proof of Proposition 5. To prove the first item, we need to establish that all decision variables are positive, i.e., $A_d^{N*} > 0$, $\varepsilon > 0$, and A_d^{Y*}. This implies, in turn, $\pi_d\eta_d - \omega - c > 0$ (from (A.16)), $\pi_d\eta_d - \omega + c > 0$ (from (D.16)), and $3c - \pi_d\eta_d + \omega > 0$ (from (D.16)). It can be easily shown that

$$\max\left\{0, \frac{(\pi_d\eta_d - \omega)}{3}\right\} < c < \pi_d\eta_d - \omega. \quad \text{(E.1)}$$

Straightforward comparisons, using the values in (A.16), (B.9), and (D.16), lead to the results:

$$A_s^{N*} = A_s^{Y*} < A_s^{I*}, \quad \text{(E.2)}$$
$$A_p^{N*} = A_p^{Y*} < A_p^{I*}, \quad \text{(E.3)}$$
$$A_d^{N*} < A_d^{Y*} < A_d^{I*}, \quad \text{(E.4)}$$

$$\begin{aligned}
J_s^{Y*} - J_s^{N*} &= e^{-\rho t}\left(V_s^{Y*} - V_s^{N*}\right)\\
&= \frac{e^{-\rho t}\gamma^2(\pi_s\eta_s+\omega)(3c - \pi_d\eta_d + \omega)}{2\mu_d\rho(\rho+\varphi)^2} > 0,
\end{aligned} \quad \text{(E.5)}$$

$$\begin{aligned}
J_p^{Y*} - J_p^{N*} &= e^{-\rho t}\left(V_p^{Y*} - V_p^{N*}\right)\\
&= \frac{e^{-\rho t}\gamma^2(3c - \pi_d\eta_d + \omega)^2}{8\mu_d\rho(\rho+\varphi)^2} > 0,
\end{aligned} \quad \text{(E.6)}$$

$$\begin{aligned}
J_d^{Y*} - J_d^{N*} &= e^{-\rho t}\left(V_d^{Y*} - V_d^{N*}\right)\\
&= \frac{e^{-\rho t}\gamma^2(\pi_d\eta_d - \omega - c)(3c - \pi_d\eta_d + \omega)}{4\mu_d\rho(\rho+\varphi)^2} > 0,
\end{aligned} \quad \text{(E.7)}$$

From the above inequalities ((E.5)-(E.7)), we get

$$J_{sc}{}^{Y*} - J_{sc}{}^{N*} = \left(J_s{}^{Y*} + J_p{}^{Y*} + J_d{}^{Y*}\right) - \left(J_s{}^{N*} + J_p{}^{N*} + J_d{}^{N*}\right) > 0. \tag{E.8}$$

This proves Proposition 5 and completes the proof. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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