PERIODIC ORBITS IN THE RESTRICTED THREE BODY PROBLEM WITH RADIATION PRESSURE

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Abstract. This paper deals with the Restricted Three Body Problem (RTBP) in which we assume that the primaries are radiation sources and the influence of the radiation pressure on the gravitational forces is considered; in particular, we are interested in finding families of periodic orbits under theses forces.

By means of some modifications to the method of numerical continuation of natural families of periodic orbits, we find several families of periodic orbits, both in two and three dimensions. As starters for our method we use some known periodic orbits in the classical RTBP.

Key words: Restricted three body problem, radiation pressure, periodic orbits

1. Introduction

The restricted three-body problem is one of the most widely studied in celestial mechanics. Its applications span the solar system dynamics, the lunar theory, the motion of spacecrafts, stellar dynamics, etc.. This problem concerns with the motion of a particle of *infinitesimal* mass which is attracted by two primaries which are moving in a non–perturbed Keplerian orbit each around the other.

Nevertheless, in some occasions, the model of three point mass is not sufficient to describe the dynamics of the problem; some additional forces must be introduced and their effects taken into account. In some cases, it is necessary to consider the primaries as rigid bodies and the non sphericity of them gives rise to some differences with respect to the results in the classical RTBP (see for instance the works of Bhatnagar and collaborators (Bhatnagar and Chawla (1977); Bhatnagar and Hallan (1983); Sharma (1981); Sharma and Subba Rao (1986)) and (Elipe and Ferrer (1985); Elipe and Arribas (1986); Elipe, 1992)). In other cases, the so called magnetic-binary problem, that is the motion of a charged particle in the field of two rotating dipoles (Kalvouridis (1994); Mavraganis (1981)) is considered.

Other studies are focussed on the stellar problem, taking the primaries as radiation sources, and the influence of the radiation pressure on the effective gravitational potential is investigated (Kunitsyn and Tureshbaev (1985); Schuerman (1980); Simmons, McDonald and Brown (1985)). This is precisely the problem here considered: the motion of a particle in the field of two luminous massive bodies. As it was pointed out by (Simmons, McDonald and Brown (1985)), this model may be applied to investigate the accumulation of matter around binaries, particularly where the stars are of luminous late type. Even more, new directions are open

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with the recent discovery of binary pulsars (Wolszczan (1985); Wolszczan (1994); Malhotra (1993)).

In this paper, we center our attention in computing families of periodic orbits. In particular, we choose the masses of both primaries to be equal, and only one radiating primary; the reason for choosing both masses equal, is that this case is quite common in binary stars, and besides, this case has been extensively studied for the classic RTBP (see for instance (Belbruno *et al.*, 1994) and references therein). The method employed is the numerical continuation with respect to one parameter, namely the radiation pressure coefficient. The algorithm used is the one described in (Lara *et al.*, 1995), based on the one proposed by Deprit and Henrard (Deprit and Henrard, 1967) that computes the intrinsic variational equations. For computing the families of three dimensional periodic orbits, we use an extension of the one of Deprit and Henrard that will be published elsewhere.

2. Equations of the motion

Let us consider two stars O_1 and O_2 with masses m_1 and m_2 that moves one around the other under a mutual force that is proportional to a function of the mutual distance and in the direction of the line joining them. The force will be specified later on. We will assume that these two stars —called primaries— move on circular orbits around their mutual center of mass O with a constant angular velocity n. Besides, let us consider a point mass O_3, whose mass m_3 is *infinitesimal*, that is to say, it does not affect the motion of the two stars but it is attracted by both primaries.

The potential function acting on O_3 may be written as

$$\mathcal{V} = \mathcal{V}^{13} + \mathcal{V}^{13} = -f\, m_1 m_3 G_1 - f\, m_2 m_3 G_2,$$

with f the gravitational constant and G_i functions depending on the distance $r_i = \|\boldsymbol{x}_i\| = \|\overline{O_i O_3}\|$, that is, $G_i = G_i(r_i)$; these functions will be specified later on.

One of the authors (Elipe, 1992) introduced a generic notation for handling this general problem. This is the one that we follow here. For details, see (Elipe, 1992). By introducing the functions

$$g_i = \frac{\partial G_i}{\partial (1/r_i)}, \qquad (1)$$

the gradient with respect to an inertial frame is

$$\boldsymbol{\nabla}_{\boldsymbol{X}} G_i = \frac{\partial G_i}{\partial (1/r_i)} \boldsymbol{\nabla}_{\boldsymbol{X}} (1/r_i) = -g_i \frac{\boldsymbol{r}_i}{r_i^3}$$

and hence, the equations of the motion are

$$\ddot{\boldsymbol{r}} = -f\, m_1 g_1 \frac{\boldsymbol{r}_1}{r_1^3} - f\, m_2 g_2 \frac{\boldsymbol{r}_2}{r_2^3}. \qquad (2)$$

Let us consider now a synodic frame $Oxyz$, such that the Ox–axis is in the direction of one of the primaries OO_2, the Oz–axis in the normal direction to the orbital plane of the primaries and the Oy–axis the complement to the right-oriented frame. In this reference system, the coordinates of the primaries O_1 and O_2 are $(x_1, 0, 0)$ and $(x_2, 0, 0)$ respectively. Let (x, y, z) be the coordinates of the particle O_3; by virtue of the moving frame theorem, the equations of motion become

$$
\begin{aligned}
\ddot{x} - 2n\dot{y} &= n^2 x - f\, m_1 g_1 \frac{x - x_1}{r_1^3} - f\, m_2 g_2 \frac{x - x_2}{r_2^3}, \\
\ddot{y} + 2n\dot{x} &= n^2 y - f\, m_1 g_1 \frac{y}{r_1^3} - f\, m_2 g_2 \frac{y}{r_2^3}, \\
\ddot{z} &= -f\, m_1 g_1 \frac{z}{r_1^3} - f\, m_2 g_2 \frac{z}{r_2^3}.
\end{aligned} \tag{3}
$$

As it is usual in the classical RTBP, the units are chosen in such a way that the distance between the primaries, the mean motion of the Keplerian orbit of the primaries and the Gauss constant ($f(m_1 + m_2)$) are equal to one. With these units, and putting $fm_2 = \mu$, equations (3) are

$$
\begin{aligned}
\ddot{x} - 2\dot{y} &= x - (1 + \mu) g_1 \frac{x + \mu}{r_1^3} - \mu g_2 \frac{x - 1 + \mu}{r_2^3}, \\
\ddot{y} + 2\dot{x} &= y - (1 + \mu) g_1 \frac{y}{r_1^3} - \mu g_2 \frac{y}{r_2^3}, \\
\ddot{z} &= -(1 + \mu) g_1 \frac{z}{r_1^3} - \mu g_2 \frac{z}{r_2^3},
\end{aligned} \tag{4}
$$

with $r_1^2 = (x + \mu)^2 + y^2 + z^2$ and $r_2^2 = (x - 1 + \mu)^2 + y^2 + z^2$.

There is an integral of these equations, the Jacobian constant

$$
C = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - (x^2 + y^2) - 2(1 - \mu)G_1 - 2\mu G_2.
$$

The general formulation obtained by the functions g_i (1) has the advantage that the same formula is valid for different types of forces. Thus, the classical case (primaries mass points with only gravitational potential) is recovered for $g_i = 1$. The influence of the non sphericity of some of the primaries (rigid body) is obtained for $g_i = 1 + J_i/r_i^2$. The case we are dealing with in this note —primaries considered mass points taking into consideration the radiation pressure— is formulated by putting $g_i = (1 - \beta_i)$, where β_i is the ratio of the radiation pressure force to the gravitational force (Schuerman (1980)).

3. Natural families of planar periodic orbits

Let us consider the two degrees of freedom system

$$
\begin{aligned}
\ddot{x} &= 2A(x, y; \sigma)\, \dot{y} + W_x(x, y; \sigma), \\
\ddot{y} &= -2A(x, y; \sigma)\, \dot{x} + W_y(x, y; \sigma),
\end{aligned} \tag{5}
$$

where A and W are two functions depending on the coordinates and on one parameter σ. This system has the Jacobian integral

$$C = 2W - (\dot{x}^2 + \dot{y}^2).$$

Each manifold $C = -2h$ is determined by the initial conditions. By putting $U = W(x, y; \sigma) + h$, the system (5) together with the Jacobian integral can be written as

$$\begin{aligned} \ddot{x} &= 2A(x, y; \sigma)\dot{y} + U_x(x, y; \sigma), \\ \ddot{y} &= -2A(x, y; \sigma)\dot{x} + U_y(x, y; \sigma), \\ \mathcal{I} &= \tfrac{1}{2}(\dot{x}^2 + \dot{y}^2) - U(x, y; \sigma) \equiv 0. \end{aligned} \tag{6}$$

Let us assume that $\boldsymbol{\xi} = \boldsymbol{\xi}(t, \sigma_0)$—solution of the system (6) for the initial conditions $\boldsymbol{\xi}_0 = (x_0, y_0, \dot{x}_0, \dot{y}_0)$— is periodic with period T_0. Hence,

$$\boldsymbol{\xi}(t; \sigma_0) = \boldsymbol{\xi}(t + T_0; \sigma_0).$$

Let us choose now a variation of the parameter $\sigma = \sigma_0 + \Delta\sigma$. The question now is how to find a new set of initial conditions $\boldsymbol{\xi}_0 + \delta\boldsymbol{\xi}_0$ such that the solution of (6) for these initial conditions be periodic with period $T_0 + \Delta T$, that is,

$$\boldsymbol{\xi}(t; \sigma_0 + \Delta\sigma) = \boldsymbol{\xi}(t + T; \sigma_0 + \Delta\sigma). \tag{7}$$

The Poincaré method of continuity ensures that, under certain conditions, there exist initial conditions

$$\boldsymbol{\xi}_0 + \sum_{k\geq 1} \boldsymbol{\xi}_{0,k}\,(\Delta\sigma)^k \tag{8}$$

such that the solution of the ordinary differential equations (6) is periodic with period

$$T = T_0 + \sum_{k\geq 1} T_{0,k}\,(\Delta\sigma)^k.$$

Under the impossibility of carrying out the infinity terms of the above series, and assuming $\Delta\sigma$ small enough, we consider only the first order approximation for the initial conditions

$$\boldsymbol{\xi}_0 + \Delta\sigma\delta\boldsymbol{\xi}_0; \tag{9}$$

the corrections $\delta\boldsymbol{\xi}_0$ are solutions of the variational system

$$\begin{aligned} \delta\ddot{x} &= 2A\delta\dot{y} + (U_{xx} + 2A_x\dot{y})\delta x + (U_{xy} + 2A_y\dot{y})\delta y + U_{x\sigma} + 2A_\sigma\dot{y}, \\ \delta\ddot{y} &= -2A\delta\dot{x} + (U_{xy} - 2A_x\dot{x})\delta x + (U_{yy} - 2A_y\dot{x})\delta y + U_{y\sigma} - 2A_\sigma\dot{x}, \\ \delta\mathcal{I} &= \dot{x}\delta\dot{x} + \dot{y}\delta\dot{y} - U_x\delta x - U_y\delta y - U_\sigma \equiv 0. \end{aligned} \tag{10}$$

Since the system (10) is linear, the general solution is formed by a linear combination of particular solutions that depends on arbitrary integration constants. The desired variations will be determined by computing those values of the integration constants that fulfil the periodicity condition (7). Notice from (7) that the unknown ΔT is an implicit variable; we overcome this inconvenience by expanding (7) up to the first order.

The Cartesian formulation of the variational equations (10) mix secular terms produced by displacements along the orbit with periodic terms. An alternative formulation can be given by using intrinsic coordinates: the tangent p and the normal n displacements obtained from the rotation

$$\begin{aligned} \delta x &= p\cos\phi - n\sin\phi, \;\; \delta\dot{x} = \dot{p}\cos\phi - \dot{n}\sin\phi - \dot{\phi}\,\delta y, \\ \delta y &= p\sin\phi + n\cos\phi, \;\; \delta\dot{y} = \dot{p}\sin\phi + \dot{n}\cos\phi + \dot{\phi}\,\delta y. \end{aligned} \tag{11}$$

When using intrinsic coordinates, the variational differential equations result to be separable (Deprit and Henrard, 1967): the normal displacements are obtained from the generalized Hill equation

$$\ddot{n} + \Theta n = -2U_\sigma \frac{A+\dot{\phi}}{V} - 2A_\sigma V + U_{y\sigma}\cos\phi - U_{x\sigma}\sin\phi, \tag{12}$$

where

$$V^2 = \dot{x}^2 + \dot{y}^2, \qquad \tan\phi = \dot{y}/\dot{x}, \tag{13}$$

and

$$\Theta = \frac{\ddot{V}}{V} + 2(A+\dot{\phi})^2 + 2A^2 - U_{xx} - U_{yy} - 2V(A_x\sin\phi - A_y\cos\phi). \tag{14}$$

The next step is the computation of the tangent displacement p by integrating the quadrature

$$\frac{d}{dt}\left(\frac{p}{V}\right) = 2\,\frac{A+\dot{\phi}}{V}n + \frac{U_\sigma}{V^2}. \tag{15}$$

The corrections computed from (9) are tangent approximations, hence the algorithm must consist of two steps: the tangent predictor followed by an isoenergetic corrector (see (Deprit and Henrard, 1967)).

Let us apply the method above exposed to the problem described in section 2. To have planar orbits, we need to make zero the third equation of the system (4). The case for which we compute the families of periodic orbits is the one in which both primaries have equal masses ($\mu = 1/2$), and only one radiating primary (O_2); hence $g_1 = 1$, $g_2 = (1-\beta)$. The parameter to vary is $\sigma \equiv \beta$. The corresponding variational equations are obtained from the Table 1 (note that in our case, $A = 1$).

TABLE I
Partial derivatives of the effective potential function U.

$$U_x = x \quad (1-\mu)\frac{x+\mu}{r_1^3} - (1-\beta)\mu\frac{x+\mu-1}{r_2^3}$$

$$U_y = y\left[1 - \frac{1-\mu}{r_1^3} - (1-\beta)\frac{\mu}{r_2^3}\right], \qquad U_z = -z\left[\frac{1-\mu}{r_1^3} + (1-\beta)\frac{\mu}{r_2^3}\right]$$

$$U_\beta = -\frac{\mu}{r_2}, \qquad U_{\beta,x} = \frac{\mu}{r_2^3}(x+\mu-1), \qquad U_{\beta,y} = \frac{\mu}{r_2^3}y, \qquad U_{\beta,z} = \frac{\mu}{r_2^3}z$$

$$U_{x,x} = 1 + \frac{1-\mu}{r_1^5}[3(x+\mu)^2 - r_1^2] + (1-\beta)\frac{\mu}{r_2^5}[3(x+\mu-1)^2 - r_2^2]$$

$$U_{x,y} = U_{y,x} = 3y\left[\frac{1-\mu}{r_1^5}(x+\mu) + (1-\beta)\frac{\mu}{r_2^5}(x+\mu-1)\right]$$

$$U_{x,z} = U_{z,x} = 3z\left[\frac{1-\mu}{r_1^5}(x+\mu) + (1-\beta)\frac{\mu}{r_2^5}(x+\mu-1)\right]$$

$$U_{y,y} = 1 + \frac{1-\mu}{r_1^5}[3y^2 - r_1^2] + (1-\beta)\frac{\mu}{r_2^5}[3y^2 - r_2^2]$$

$$U_{y,z} = U_{z,y} = 3yz\left[\frac{1-\mu}{r_1^5} + (1-\beta)\frac{\mu}{r_2^5}\right], \qquad U_{z,z} = \frac{1-\mu}{r_1^5}[3z^2 - r_1^2] + (1-\beta)\frac{\mu}{r_2^5}[3z^2 - r_2^2]$$

To begin with, we need an initial periodic orbit. This is chosen from the classical RTBP ($\beta = 0$); with this starter we compute the family for small variations of β. In Figures 1 and 2 we present several periodic orbits corresponding to two different families. The corresponding initial conditions for each orbit —and the value of β— appear in Tables 2 and 3.

4. 3-D families

For computing three dimensional families of periodic orbits, we make an extension of the algorithm given by Deprit and Henrard (Deprit and Henrard, 1967). The main feature of this extension consists in formulating the variational equations in the Frenet frame $(\boldsymbol{t}, \boldsymbol{n}, \boldsymbol{b})$, where $\boldsymbol{t}$ is the tangent, $\boldsymbol{n}$ the normal and $\boldsymbol{b}$ the binormal vectors of the orbit. As it is well known, these vectors are

$$\boldsymbol{t} = \dot{\boldsymbol{x}}/V, \qquad \boldsymbol{n} = \dot{\boldsymbol{t}}/N, \qquad \boldsymbol{b} = \boldsymbol{t} \times \boldsymbol{n},$$

with $V^2 = \dot{\boldsymbol{x}} \cdot \dot{\boldsymbol{x}}$ and $N^2 = \dot{\boldsymbol{t}} \cdot \dot{\boldsymbol{t}}$. An intrinsic variation is

$$\boldsymbol{s} = p\,\boldsymbol{t} + q\,\boldsymbol{n} + r\,\boldsymbol{n}.$$

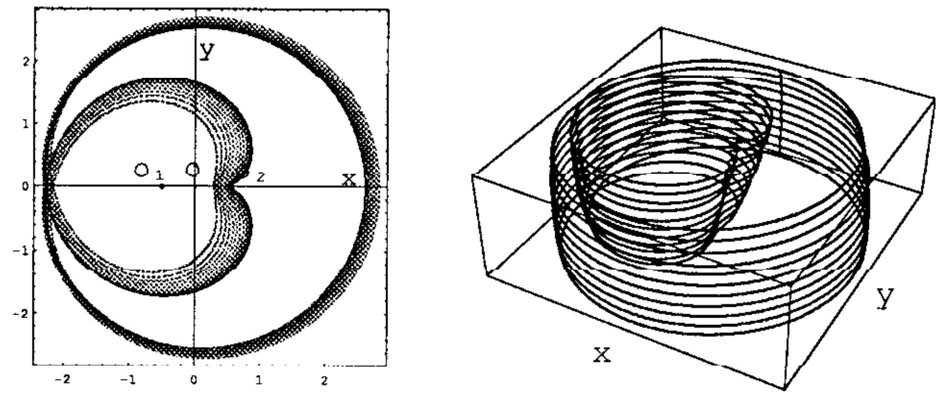

Fig. 1. The family of periodic orbits for the Jacobian integral $h = -0.14897239$, and variations of β from $\beta = 0$ till $\beta = 0.986$ and $\delta\beta = 0.1$. The right figure is a spatial representation of the same set of orbits; the vertical axis represents β.

TABLE II

Several periodic orbits of the family presented in Figure 1 for $h = -0.14897239$.

β	T	Trace
0.0	0.1044611108660456E+02	0.3822153997099804E+02
0.1	0.1070172013219846E+02	0.5742991976362558E+02
0.2	0.1091861983180412E+02	0.7877942498646655E+02
0.3	0.1111613216668850E+02	0.1037624867333485E+03
0.4	0.1130400877313352E+02	0.1344319449796839E+03
0.5	0.1148877305694259E+02	0.1742951152990996E+03
0.6	0.1167602660316857E+02	0.2302916965480838E+03
0.7	0.1187195100933765E+02	0.3186898086851346E+03
0.8	0.1208539221579756E+02	0.4888860581472264E+03
0.9	0.1233378082533988E+02	0.9919428359728460E+03
0.986	0.1262371447640814E+02	0.7268460161558314E+04

β	x_0	y_0	$\dot{x}_0$
0.0	0.1807988571131529E+00	0.2658263191705089E+01	0.2745718742061957E+01
0.1	0.1807767909534300E+00	0.2678057761574384E+01	0.2757228143972670E+01
0.2	0.1807995814686417E+00	0.2685745453338287E+01	0.2757616160628099E+01
0.3	0.1807951549726286E+00	0.2685443316541738E+01	0.2750545269799129E+01
0.4	0.1807226536826620E+00	0.2678957662978194E+01	0.2737601543522081E+01
0.5	0.1805502674655450E+00	0.2667173063771944E+01	0.2719537699281033E+01
0.6	0.1802470712670701E+00	0.2650481485311404E+01	0.2696654823880121E+01
0.7	0.1797778243600812E+00	0.2628926901761657E+01	0.2668927197288515E+01
0.8	0.1790970074637502E+00	0.2602209226312452E+01	0.2635996989486309E+01
0.9	0.1781377416068378E+00	0.2569525571175686E+01	0.2597012717347751E+01
0.986	0.1769986552205803E+00	0.2535174972786400E+01	0.2557151270760333E+01

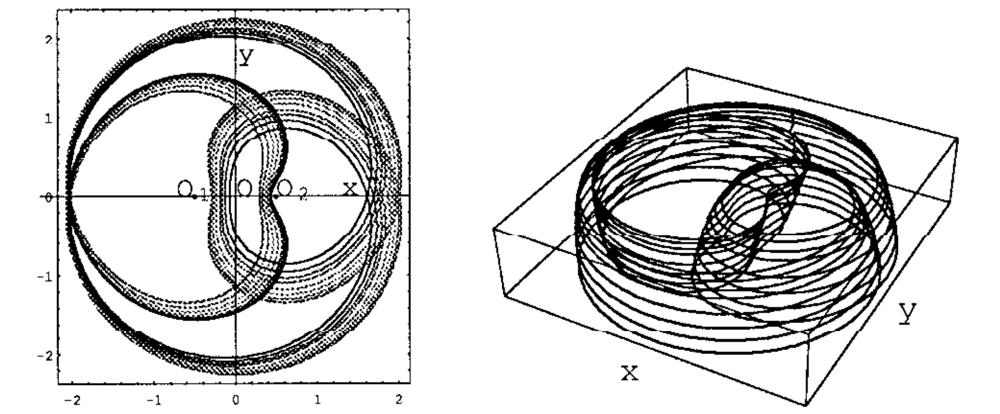

Fig. 2. The family of periodic orbits for the Jacobian integral $h = -0.27279945$, and variations of β from $\beta = 0$ till $\beta = 0.537$. The right figure is a spatial representation of the same set of orbits: the vertical axis represents β.

TABLE III

Several periodic orbits of the family presented in Figure 2. The value of $h = -0.27279945$.

β	T	Trace
0.0	0.1494816481714137E+02	0.6188380629984958E+03
0.099	0.1515637579861915E+02	0.8120723863623102E+03
0.21	0.1533733436551117E+02	0.9804795169153184E+03
0.3	0.1544687308727921E+02	0.1052050814120363E+04
0.399	0.1551583303720730E+02	0.1002491121777919E+04
0.45	0.1551412773178408E+02	0.8793267669154791E+03
0.501	0.1545198506391003E+02	0.6187755693006095E+03
0.537	0.1525584143212497E+02	0.1811784593415472E+03

β	x_0	y_0	$\dot{x}_0$
0.0	-0.2703535202131601E+00	0.2228189299382710E+01	0.2296266524432066E+01
0.099	-0.2690571464753951E+00	0.2217860181527281E+01	0.2280599330513873E+01
0.21	-0.2664917693148786E+00	0.2195799442952172E+01	0.2253077929615262E+01
0.3	-0.2637308598863723E+00	0.2170060236156629E+01	0.2223289994780886E+01
0.399	-0.2599884680685741E+00	0.2131822567937924E+01	0.2181166523493134E+01
0.45	-0.2576830098628738E+00	0.2106121672926874E+01	0.2153905670760668E+01
0.501	-0.2548930500571476E+00	0.2072417468486886E+01	0.2119400522003604E+01
0.537	-0.2518278131800796E+00	0.2031352149271426E+01	0.2079777506305585E+01

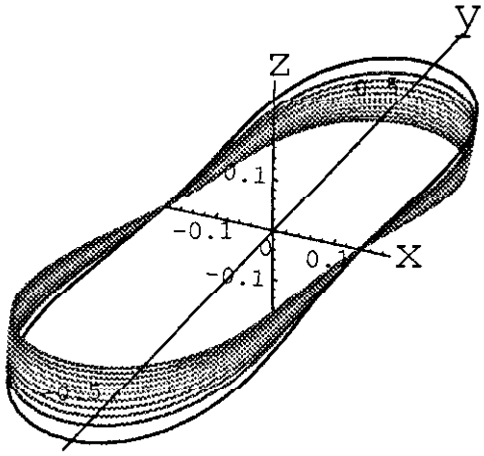

Fig. 3. Several 3-D periodic orbits of the family corresponding to a value of the Jacobian constant $h = -1.33815281$.

TABLE IV
Several 3-D periodic orbits of the family presented in Figure 3.

β	x_0	y_0	z_0
0.0003	0.1638871692679212E+00	-0.2510362681618084E-08	0.7005048919400680E-09
0.0021	0.1653678717992377E+00	-0.2557209555220652E-08	0.6330022239882382E-09
0.0036	0.1665958769975350E+00	-0.2674621274456612E-08	0.5847132335604077E-09
0.0048	0.1675744820870033E+00	-0.3191755856141176E-08	0.6157965335568116E-09
0.0059	0.1684686161461838E+00	-0.1507369601708164E-07	0.2508729056958925E-08
0.0069	0.1692790734294930E+00	-0.2024547203645886E-07	0.2803378619276346E-08
0.0079	0.1700872811034856E+00	-0.1146168880413606E-06	0.1189589651047138E-07
0.0089	0.1708932660742574E+00	-0.1159858110111323E-06	0.5789087144662981E-08

β	$\dot{y}_0$	$\dot{z}_0$	T
0.0003	-0.1303399906260374E+01	0.3637044861399464E+00	3.7173529930360
0.0021	-0.1315310325495506E+01	0.3255891893476048E+00	3.7170517495484
0.0036	-0.1325231216349784E+01	0.2897160756048663E+00	3.7168002723863
0.0048	-0.1333165665236091E+01	0.2572127822288803E+00	3.7165988007882
0.0059	-0.1340437566303580E+01	0.2230901524732224E+00	3.7164138913028
0.0069	-0.1347047571217604E+01	0.1865248622751126E+00	3.7162456025896
0.0079	-0.1353657066609580E+01	0.1404938044559164E+00	3.7160771331593
0.0089	-0.1360266306763544E+01	0.6789365573312622E−01	3.7159084824835

The separability of the tangent variations when formulated in the Frenet frame was demonstrated by (Deprit, 1981). Indeed, given the conservative system

$$\ddot{\boldsymbol{x}} + \text{curl}\boldsymbol{A} \times \dot{\boldsymbol{x}} - \boldsymbol{\nabla}_{\boldsymbol{x}} U = 0,$$

that has the integral

$$C = \dot{\boldsymbol{x}} \cdot \dot{\boldsymbol{x}} - 2U.$$

the intrinsic variations can be computed by solving a coupled system involving only the normal and binormal displacements, and computing the quadrature

$$\frac{d}{dt}\left(\frac{p}{V}\right) = \left(2\frac{N}{V} + \frac{\text{curl}\boldsymbol{A} \cdot \boldsymbol{b}}{V}\right) q - \frac{\text{curl}\boldsymbol{A} \cdot \boldsymbol{n}}{V} r + \frac{U_\sigma}{V^2},$$

which shows the sepparability of the tangent displacements. For details, cfr. (Deprit, 1981). Similarly to the two degrees of freedom case, the algorithm consists of a tangent predictor and an isoenergetic corrector.

We now apply the procedure to the spatial motion defined by the system (4). The variational system is obtained from Table 1.

In Figure 3 we present several orbits of a family. Similarly to the planar orbits, the starting periodic orbit for the algorithm is chosen from the classic RTBP. The corresponding initial conditions are showed in Table IV.

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