PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Doug Hensley, Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Itshak Borosh, Paul Bracken, Ezra A. Brown, Randall Dougherty, Dennis Eichhorn, Tamás Erdélyi, Kevin Ford, Zachary Franco, Christian Friesen, Ira M. Gessel, Jerrold R. Griggs, Jerrold Grossman, Kiran S. Kedlaya, Andre Kündgen, Frederick W. Luttman, Vania Mascioni, Frank B. Miles, Richard Pfiefer, Cecil C. Rousseau, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Walter Stromquist, Daniel Ullman, Charles Vanden Eynden, and Fuzhen Zhang.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted solutions should arrive at that address before March 31, 2004. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11040. *Proposed by Yongge Tian, Queen's University, Kingston, Canada.* Let A and B be $n \times n$ complex matrices such that $A = A^2 = A^*$ and $B = B^2 = B^*$. That is, A and B are both idempotent and Hermitian. Show that

$$\mathrm{range}[(AB)^2 - (BA)^2] = \mathrm{range}(ABA - BAB) = \mathrm{range}(AB - BA).$$

11041. *Proposed by Oscar Ciaurri, Universidad de La Rioja, La Rioja, Spain.* Let w be a real number with $-1 < w < 1$. Let

$$\alpha(2m+1) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^{2m+1}} \quad , \beta(2m) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^{2m}},$$

and

$$g_1(w) = \sum_{m=1}^{\infty} \alpha(2m+1) w^{2m} \quad , g_2(w) = \sum_{m=1}^{\infty} \beta(2m) w^{2m-1} .$$

Show that

$$g_1(w) + \frac{1}{2}\log 2 = \frac{1}{2}\left(g_1\left(\frac{w+1}{2}\right) + g_1\left(\frac{w-1}{2}\right)\right)$$

and

$$g_2(w) = \frac{1}{2}\left(g_1\left(\frac{w+1}{2}\right) - g_1\left(\frac{w-1}{2}\right)\right).$$

11042. *Proposed by M. N. Deshpande, Institute of Science, Nagpur, India, and Kavita Laghate, S.N.D.T. Women's University, Mumbai, India.* Let n and k be positive integers