Proceedings of the Edinburgh Mathematical Society (1994) 37, 519–520 ©

CORRIGENDUM ET ADDENDUM: THE FRATTINI SUBALGEBRA OF A BERNSTEIN ALGEBRA

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(Received 5th March 1993)

In a previous paper it is supposed that if A is a Bernstein algebra, every maximal subalgebra, M, verifies that $\dim M = \dim A - 1$. This is not true in general. Therefore Proposition 2 in this paper is not correct. However other results there, where this assertion was used, are correct but their proofs need some modifications now.

1991 *Mathematics subject classification*: 17 DXX

1. Maximal subalgebras of Bernstein algebras

If we take the commutative algebra A over a field K spanned by $\{e,u_1,u_2,v_1,v_2\}$ such that $eu_i = 1/2u_1$, $u_1v_1 = u_1$, $u_1v_2 = -u_2$, $u_2v_i = u_i$, $i = 1,2$ and the other products equals to zero, we have a Bernstein algebra. In this algebra, the subalgebra M spanned by $\{e,v_1,v_2\}$ is maximal subalgebra. So if M is maximal subalgebra of a Bernstein algebra A, we do not always have $\dim M = \dim A - 1$. But we will see that if A is also genetic then $\dim M = \dim A - 1$ for every M maximal subalgebra.

Lemma 1. *Let A be a Bernstein algebra, e a nonzero idempotent in A such that $A = Ke \oplus U_e \oplus V_e$, and M a maximal subalgebra of A. Then U_e^2, U_e^3, $(U_e^2)^2 \leqq M$.*

Proof. Let $N = U_e + U_e^2$. We have that $B = Ke + N$ is a subalgebra of A and a Bernstein algebra. From [1] it is known that a Bernstein algebra A with $A^2 = A$ is genetic. Since $B^2 = B$, we have that N is nilpotent, and from [3] $F(N) = N^2$. But $N^2 = U_e^2 + U_e^3 + (U_e^2)^2$ is an ideal of A because of [4] (or checking it directly). Therefore, using [3], we have $N^2 \leqq F(A)$, that is U_3^2, U_e^3, $(U_e^2)^2 \leqq M$ for every maximal subalgebra M.

Lemma 2. *Let M be a maximal subalgebra of a Bernstein algebra A and let e be an idempotent in M. Then either $V_e \leqq M$ or $U_e \leqq M$.*

Proof. Clearly $M = Ke + U'_e + V'_e$ with $U'_e \leqq U_e$, $V'_e \leqq V_e$. Now $M + U_e = M$ or $M + U_e \leqq A$. The former implies that $U'_e = U_e$; the latter implies that $V'_e = V_e$.

Now it is easy to prove the following results.

Theorem 3. *Let A be a genetic Bernstein algebra. If M is a maximal subalgebra of A, then* $\dim M = \dim A - 1$. *Therefore a vector subspace M of A is a maximal subalgebra if and only if*
either (i) $M = \operatorname{Ker} w$,
or (ii) *M has a nonzero idempotent e such that M is one of the following subalgebras*
(a) $M = K.e \oplus U_e \oplus V'_e$ *with* $V'_e \leqq V_e$ *such that* $\dim V'_e + 1 = \dim V_e$ *and* $U_e^2 \leqq V'_e$; *in this case M is an ideal,*
(b) $M = K.e \oplus U'_e \oplus V_e$ *with* $U'_e \leqq U_e$, $\dim U'_e + 1 = \dim U_e$, $U_e V_e + V_e^2 \leqq U'_e$,

Proposition 4. *Let A be a Bernstein algebra and M a vector subspace of A. Then M is a maximal subalgebra if M is one of the following subspaces*:

(i) $M = \operatorname{Ker} w$,

(ii) *M has a nonzero idempotent e such that*

(a) $M = K.e \oplus U_e \oplus V'_e$ *with* V'_e *such that* $\dim V'_e + 1 = \dim V_e$ *and* $U_e^2 \leqq V'_e$. *In this case M is an ideal.*
(b) $M = K.e \oplus U'_e \oplus V_e$ *with* $U'_e \leqq U_e$, $\dim U'_e + 1 = \dim U_e$, $U'_e V_e + V_e^2 \leqq M$
(c) $M = K.e \oplus U'_e \oplus V_e$ *with* $U'_e \leqq U_e$, *such that* $\dim U'_e + 1 < \dim U_e$, $U'_e . V_e + V_e^2 \leqq U'_e$.

Theorem 5. *Let A be a Bernstein algebra. Then $F(A)$ is an ideal.*

Proof. Let $N = U_{e^\cdot} + U_e^2$; then from the proof of Lemma 1, $N^2 \leqq F(A)$. Since $F(A/N) = 0$, then $F(A) \leqq N$. Now suppose that $AF(A) \nleqq F(A)$. Then there is a maximal subalgebra M of A with $AF(A) \nleqq M$. But $AF(A) \leqq AN \leqq N$, so $N \nleqq M$ and $A = M + N$. Thus $AF(A) = MF(A) + NF(A) \leqq M + N^2. \leqq M$, a contradiction.

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