Article

References-enriched Concept Map: a tool for collecting and comparing disparate definitions appearing in multiple references

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Abstract
Finding and sharing a common vocabulary is a critical task for the development of any area of knowledge. However, it is very common to find heated debate in the literature on the meaning of particular terms. Different authors propose different definitions, some of them even contradictory. This situation, while enriching the scientific process, may hinder the understanding of fundamental concepts regarding a certain subject. To address this problem, we propose a technique called References-enriched Concept Maps (RCM), inspired by concept maps. RCM can be used to compare definitions and therefore improve the understanding of terms, keeping track of the publications in which the different definitions were proposed. We present a method of RCM construction as well as different metrics for analyzing them. An analysis carried out using the proposed metrics allows one to find answers while it also raises new questions about the discussed concepts.

Keywords
Knowledge representation; concept map; bibliographic reference; research support; modelling concepts; RCM

1. Introduction

The existence of a certain degree of consensus on the true meaning of the terms associated with a particular subject under study (whether in research or in other fields of knowledge) is very important to avoid ambiguity and to hold a discussion focused more on the real problem than on the terms used. However, debates concerning the true meaning of a particular term appear very frequently in the relevant literature. These debates usually result in a multitude of references that include very different, or even contradictory, definitions. When the quantity and diversity of definitions is very high it is quite difficult to pinpoint the relevant concepts and the most widely accepted definitions, as well as to understand why differences appear. Modelling is a particular case that illustrates the related problem since there is ample and diverse literature on the meaning of various terms (model, modelling language, metamodel, etc.), but, after more than a decade of studies and discussions on the matter, there is still no consensus on their true meaning [1]. The main objective of this paper is to comprehensively explain a tool, called References-enriched Concept Map (RCM), aimed at clarifying those issues. We presented RCM in a previous work where we addressed a discussion on different modelling issues [2]. To support the in-depth review of 200 references about modelling we dealt with, we needed a tool that graphically summarized the different definitions of each of the terms we covered. In [2], because of space limitations, we could not explain in detail neither all the technical features of RCM nor its methodological aspects rooted in Concept Maps [3,4] (even the name that we gave to the tool in [2] is slightly different from the one that we are using now).

Concept Maps are graphical tools for structuring and representing knowledge. Concept maps feeds off the ideas of the constructivist learning movement and the Ausubel's theory of meaningful learning. Within this theoretical framework, concept maps are mainly used for stimulating the establishment of relationships among concepts and for the generation of ideas, constituting an excellent breeding ground for the construction of new and more elaborated knowledge by the map developer or reader. From this perspective, our approach might seem to contradict the true philosophy of concept-maps, since our aim is not to catalyze learning but to capture the knowledge (definitions) that a set of experts in a subject of study have published in a set of bibliographic references, as much as possible preserving the original statement of those definitions. Nevertheless, RCM does not relinquish its capacity to generate new knowledge, capacity that is rooted in its concept-map origin. Once done, an RCM enables the calculation of a series of metrics that help to analyze the collected definitions, not only to answer the questions that motivated its development, but to raise new discussions about the concept on which it focus.

We could have proposed to use plain concept maps for discussing and comparing the different definitions that exist in the literature concerning a certain topic, but we found that neither Concept Maps nor other knowledge representation graphical techniques (see Section 5) are really appropriate for performing that task. Concept Maps enable the linking of resources, e.g. bibliographic references, to a concept. But, when it comes to clarifying the definition of a term, the mere association of each concept in the definition with the reference where it appears is not enough. On the contrary, it is necessary to provide a way of traversing the concepts in the map to reconstruct the original definition as close as possible to the way it was enunciated in the bibliographic reference. RCM solves this problem by labelling the links along concepts not only with a linking phrase (as basic Concept Maps do) but with the different references where these definitions are included. This feature, that we call *path labelling*, brings a lot of meta-information about the concepts related to a topic. The presence of bibliographic references enables the comparison of different definitions, determining inclusion, dependence, similarity or antagonism relationships. It also facilitates the assessment of the quality of the references themselves, detecting possible lacks in comparison with the others, and assisting in determining a core set of references needed for the comprehension of a particular subject. On the other hand, the author of a RCM must show graphically that certain concepts are more relevant than others to clarify the meaning of the discussed concept. Concepts in a concept map are ranked from the most general, most inclusive, to the more specific ones [4], but they all are equally represented. RCM addresses the hierarchy of concepts in a more open way, performing what we have called a *concept layering*. This layering is realized by means of a colour-based approach in order to highlight the relevance of the concepts. The criteria applied for layering is subjective. This could be the generality criteria pointed by Novak, the semantic relevance, or could be based on the degree of scientific dissemination (for instance, the appearance of a concept in a prestigious publication, or the fact of appearing in more or fewer different references, may give clues about its importance). Concept layering is necessary for the computation of several RCM metrics we present below.

The remainder of this paper addresses the definition of an RCM, its features and the contexts in which it can be used and how. We also deal with the way of getting the most from an RCM, describing several metrics for RCM analysis. We also include a related work section in which we survey other knowledge mapping proposals, comparing them to RCMs. The paper finishes with some conclusions and directions for future work.

2. What is a References-enriched Concept Map (RCM)?

An RCM allows the visualization of a set of definitions about a term discussed in the literature, facilitating the analysis of such definitions in relation to the authors who propose them. Many definitions, whether lexical, stipulative or explanatory [5,6], can be analyzed into two elements: the term to be defined (or *defininiedum*, plural *definienda*), and another expression that explains the meaning of the term and that is equated by the definition of the definiendum (or *definiens*, plural *definientia*) [7]. Definiens is composed of more basic concepts, which constitute the building blocks of RCMs. The appearance of an RCM resembles a concept map [3,4] since it is basically a diagram showing relationships (links) between concepts. Nevertheless, RCMs enrich the links with references to the publications where the different definitions were presented. In an RCM, concepts are represented by boxes and are connected by arrows labelled with words or linking phrases. Concepts are usually nouns or adjectives and linking phrases are verbs or adverbs. As indicated in [3,4] a concept map is usually constructed from a question called *focus question* that in the case of an RCM always has the form “What is the meaning of (something)?”. Table 1 summarizes the correspondence between the main notions of general Concept Maps and RCMs.

It should be noted that a concept map, and therefore an RCM, is rarely a complete representation of the concepts and relationships in a particular knowledge domain, but rather it is a workable approximation [8]. In the particular case of RCM this implies, on the one hand, that the RCM creator must select among the available definitions in the context those he/she considers relevant and, on the other hand, that RCMs are always related to a date of last update determined by the date of the most recent reference. Therefore, an RCM is subject to new versions derived from the updates that the author makes as soon as new references to new definitions of the concept under study appear.

Table 1. Correspondence between several notions of concepts maps and References-enriched Concept Maps

Concept Maps	References-enriched Concept Maps
Concepts	The definiendum and the concepts included in its definientia

Linking phrases	Linking phrases, among which there should be some forms of the "to be" verb
Proposition	Definition, or part of a definition
Focus question	What is the meaning of (something)
Hierarchy	See Section 2.1.3 below
Context	Discussion about the specific meaning of a concept
Cross-links	Enable the comparison between different definitions (definientia) of the same concept (definiendum), and the comparison between definitions of different concepts (definienda)

2.1. Key elements of RCM

As we have explained, RCMs inherit characteristics from concept maps, but with some distinguishing peculiarities.

2.1.1 Main concept

This is the definiendum, or concept corresponding to the term, about which there are definitions in the relevant literature and to which the focus question refers.

2.1.2 Path labelling with references

Sometimes concept maps are created collaboratively. Each author provides a part of the compiled knowledge, but the resultant map does not reflect the authorship of each individual contribution. There are fields in which this attribution is relevant (collaboration between researchers, citations). We have applied this idea to RCM by labelling the links between its concepts with references to the publications where the collected definitions appear. Actually, the RCM creator does not label individual links, nor concepts, but paths on the map – i.e. sequences of concepts and linking phrases – that correspond to the definitions that the RCM collects from each reference. Certain Concept Maps tools (e.g. CMaps Tools or VUE) allow users to associate resources, such as bibliographic references, to concepts or links, but the labelling of full paths, required by RCM, is difficult to achieve using these tools. They do not support path labelling, since it is not a constituent part of the kernel of Concepts Maps. This missing functionality forces the creator of an RCM using one of these tools to manually label each link between concepts along the definition-path.

As commented, path labelling is a significant contribution of RCM. Each concept and linking phrase is attributed to the reference where it appears by means of the labels of the paths to which it belongs. This is not the case in the classical Concept Maps, in which it is impossible to distinguish among all the possible connections between concepts which of them correspond to a certain definition. Thus, references in RCM suggest a reading order on the map that helps map readers retrieve the original definitions, even when a certain definition is included in another one (we show an example of this in Figure 1). However this ordering should not be considered as mandatory, nor it is an impediment for the establishment of cross-links among the concepts of the underlying concept map. An RCM is still a concept map that may leverage new knowledge about the main concept. Path labelling is also a key element for the evaluation of the metrics we describe below.

It should be noted that an RCM is always referred to that set of references that, according to the creator of the RCM, are the most relevant definitions that include the main concept. In order to facilitate the readability of the diagram, each reference is represented by a label using some bibliographic style, standard (ACM, APA, etc.) or not. The set of references is an open list that can be progressively updated when new references with new definitions emerge. This makes an RCM a live tool that can be kept 'up to date' in order to enrich the definition of a concept according to the most recent literature on the subject.

2.1.3 Concept layering

In discussing the different definitions of a main concept, not all concepts compiled from these definitions and that are dotted around the map contribute to the same extent to the clarification of the fundamental meaning of the term. For instance, concepts attached to the main concept by some grammatical form of the verb "to be" (is, can be, etc.) define the essence of the main concept. In the most common type of definition, known as *genus and differentia definition* [7], this essence is known as *genus*, which is the kind or family of things to which the defined thing belongs, whereas the *differentia* is the set of distinguishing features that marks it off from other members of the same family.

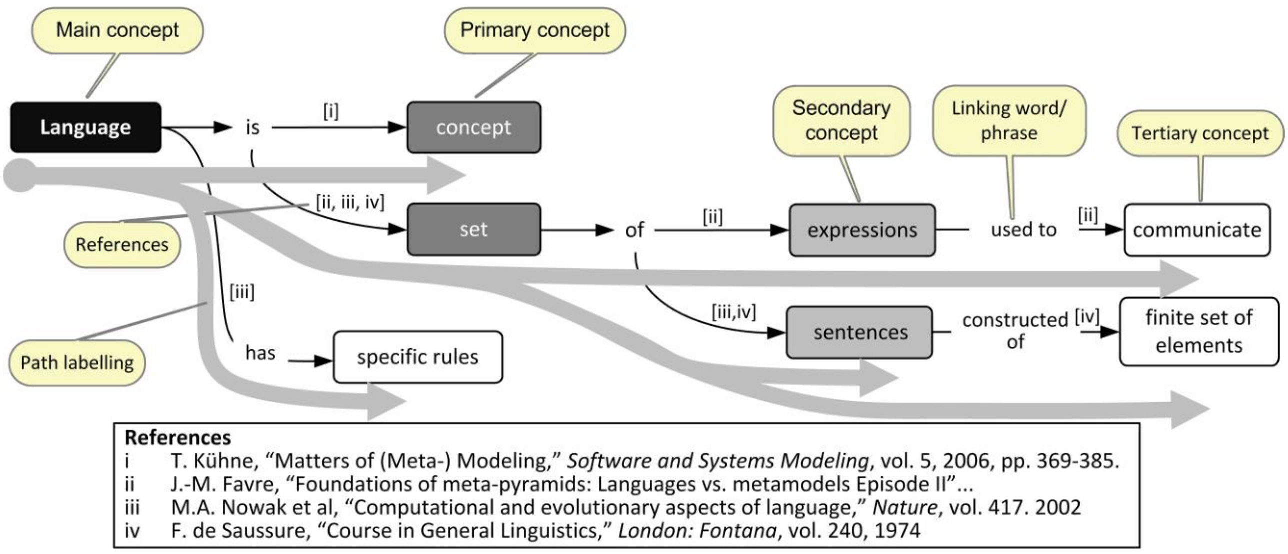

Figure 1. Basic example of RCM

Accordingly, an RCM should highlight those concepts that, in the opinion of its creator, are located in a first layer closer to the essence of the main concept. This distinguishes these concepts, which we name primary concepts, from other concepts that provide additional explanatory features of the main concept. The creator of an RCM can also classify these remainder concepts in other layers (secondary, tertiary, etc., concepts), but our recommendation is to set up no more than three or four layers. We want to stress that the creator of an RCM is free to choose the criteria for concept layering. The classification does not necessarily result from the hierarchical structure of the RCM (see an example in Figure 1), although this could be a criteria for layering. On the contrary, in the case of definitions, it seems sensible to place those concepts related to the main concept by means of the verb "to be" (the genus) in the first layer, the differentia concepts in the second layer, and devote the rest of layers for other additional features. For instance, in the definition "a table is a piece of furniture with a large flat surface supported by one or more legs", "table" is the definiendum, "piece of furniture with a large flat surface supported by one or more legs" is the definiens, "piece of furniture" is the genus and "with a large flat surface supported by one or more legs" is the differentia. From the RCM perspective, "table" must be the main concept and, surely, the RCM creator will chose "piece of furniture" as a primary concept and "surface" and "legs" as secondary concepts.

To represent the layers, RCM uses a colour graded scale, which associates a denser colour with the higher layer concepts (see Figure 1). Thus, the primary concepts allow the users of a certain RCM to get a quick visual idea on what is the essence of the definitions in that RCM. In addition, concept layering is a key element for the implementation of the various metrics that we propose below.

2.1.4 A basic example

Figure 1 presents a deliberately incomplete RCM of Language to illustrate the RCM basic elements using an example that comprises the following definitions: "Language is a concept", "Language is a set of expressions used to communicate", "Language is a set of sentences and has specific rules" and "Language is a set of sentences constructed from a finite set of elements". We have organized the concepts in three layers, "set" and "concept" being the primary concepts. The colour gradation lets the reader see at a glance which are the most important concepts for the understanding of the meaning of "language" (main concept). Although the concept "specific rules" appears in the definition following "sentences", it is a concept that complements the main concept ("language"). This is the reason why we have linked it directly to the main concept, and not to "sentences". However, despite its position on the map, we have not classified it as a primary concept. We have added to Figure 1 thick grey arrows that graphically show the path labelling, though, actually, these arrows are not part of the syntax of RCM. Finally, we want to point out that the path labelling lets the RCM contain two definitions (in references iii and iv), one included inside the other. Without references, this fact would go unnoticed.

2.2. Uses of RCMs

The main goal of RCMs is to support research and clarification of the meaning of relevant concepts in a particular context. Among others, these are the situations where their utilization can be useful:

- *Comparison of definitions*: they are particularly useful for comparing definitions when there is no consensus in the literature on the definition of a certain main concept of interest and/or there are many different definitions for it.
- *Complex definitions*: they help to visualize more clearly the most important concepts that constitute the meaning of the main concept.
- *Compact summary of definitions*: when studying a certain subject, it is usually necessary to make a summary of the main definitions found in the literature. An RCM is the perfect tool for this task, because it allows representation of all considered definitions using minimum space.
- *Analysis of bibliographic references*: RCM allows one to evaluate, among others, the quality or similarity of the definitions and, indirectly, of the references that include them.
- *Umbrella definitions*: the "big picture" that an RCM offers about the state of the art of the definitions of a specific term allows for an interesting exercise. Collecting all non-contradictory concepts proposed by the literature included in the RCM, you can make a complete and general new definition, which can be called "umbrella definition".

3. How to build an RCM?

The construction procedure of an RCM is different from that described by Novak for concept maps (see [4]). Novak suggest collecting all the concepts in a list and rank them, placing first the more general ones. This list is like a "parking lot" from where concepts are moved into the map as the map creator determines where each concept fits in. When developing a RCM, there is no such freedom. We must observe the linear appearance of the concepts in each definition, definition by definition, trying to reuse concepts already placed on the map for new definitions.

Consequently, we propose a five steps procedure to build an RCM:

1. *Collection of references*: make a list of references containing the main concept definitions.
2. *Identification and rephrasing of definitions*: identify each definition in each reference highlighting the concepts, and also the linking phrases. It should be noted, that in a well-constructed RCM, as well as in a concept map, [9] "concepts and linking phrases are as short as possible, possibly single words". Moreover, sometimes you will need to rephrase definitions adopting an equivalent statement in order to facilitate the identification of concepts e.g. if a reference defines "languages are symbol systems" and another states that "language is a system of symbols" it should be understood that both are equivalent and only one of them should be used (preferably the latter, since it uses connectives and order that facilitate the concept layering).
3. *Layout of linking phrases*: link the concepts using labelled directed arrows pointing to the linking phrases, and successively from these to the next concept.
4. *Path labelling:* label the arrows with the reference where the definition appears. You must provide a different reference number for each definition. When, occasionally, a single reference contains two definitions, you must use a different number for each definition (e.g. 14a and 14b). In any case, the definition appearing in each reference should be recoverable following the arrows labelled with such a reference.
5. *Concept layering*: choose the layering criteria and classify the concepts in layers as explained in the previous section and draw them typically from left to right. The main-concept box background must be in a dark colour, establishing a descending colour gradation to represent the layers. It is possible that the same concept is used at different layers in different definitions. In these cases, the RCM creator must choose the layer to which the concept belongs, for example based on the number of references that place the concept at one layer or another.

Following these steps, complex RCMs can be constructed in a compact way, such as that in Figure 2, which depicts the RCM of "model", which was first presented in [2]. It is made up of 21 definitions that include 10 primary, 7 secondary and 6 tertiary concepts.

Regardless of the fact that an RCM can be done with just pencil and paper, there are several applications for the construction of concept maps that, with some limitations, can be used to create an RCM. In Section 5 we survey the more relevant among these tools, analyzing their suitability for RCM creation.

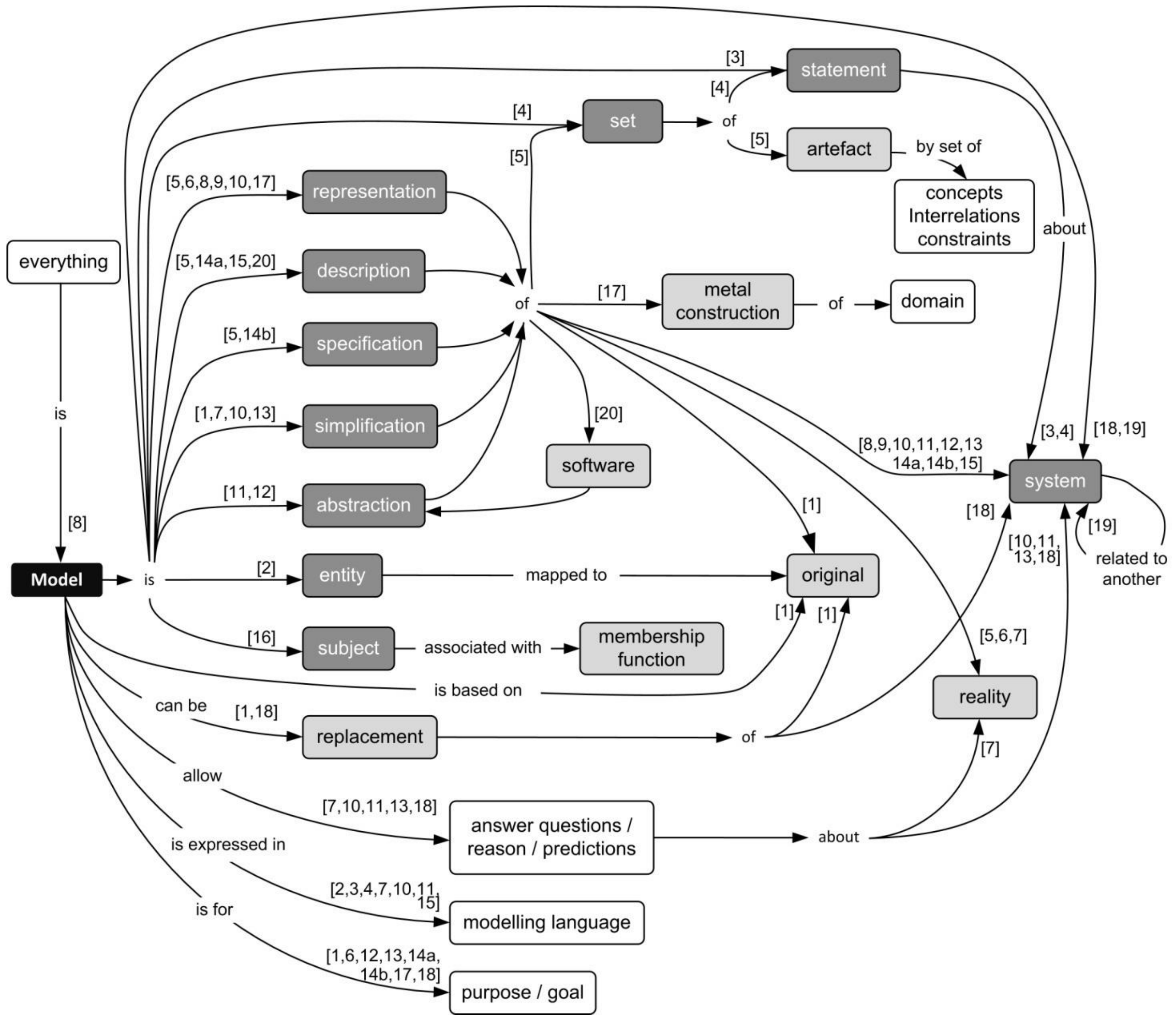

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Figure 2. RCM of 'Model' (Adapted from Rodriguez-Priego et al. Modelling Issues: a Survival Guide for a Non-expert Modeller. 2010).

4. Metrics for the analysis of RCMs

In general, Concept Maps constitute a tool that serves not only to represent knowledge but also, through their analysis, draw conclusions in different fields of learning [8,10]. In this section we describe some specific techniques that allow an RCM user to measure for example the complexity or quality of the definitions of a given main concept. The degree of complexity is proportional to several metrics that we describe below, such as the ratio concept scattering, the ratio number of links between concepts in the same layer with regard to the concepts in that layer and the number of links between concepts in non-consecutive layers. Taking for granted that there is usually agreement regarding the meaning of simple concepts, the lack of consensus in the relevant literature, indicated by a high number of concepts with a low density of references per concept or semantic field in an RCM (see *DRC* and *DRSF* below), may also be a symptom of the complexity of an RCM.

Some of these metrics could be determinable using plain Concept Maps. However, other metrics are based on the study of references and their relationship with the concepts, so that they are only applicable to RCMs. For each metric we indicate the scope (concept, layer, definition or RCM) to which the metric applies, the metric type (ratio or percentage) and the criteria applied to the metric [10,12]. Note that we make a subjective interpretation of what are the most appropriate criteria for each metric. This will not prevent you from defining other criteria or metrics to meet your needs. Sometimes, a certain criterion leads to a measurable quantitative metric, whereas on other occasions it corresponds to a property that the RCM analyst must evaluate subjectively.

4.1. Concept and semantic scattering

Concepts in a layer of an RCM can be organized into semantic fields, each one including items according, for instance, to their synonymy. Consequently, this organization results in a layer metric that we call *semantic scattering of the layer n* (SS_n), defined as the ratio:

$$SS_n = 100\frac{SF_n - 1}{D - 1}$$

where SF_n is the number of *semantic fields per layer* and *D* is the number of definitions in the RCM. This metric assesses the degree of consensus about the meaning of the main concept in the relevant literature. Thus, a measure of *SS* near 100%, especially if this occurs in first layer, indicates that disparate meanings have been found in the literature review collected in the RCM.

Similarly, we define concept scattering of the layer n (CSn) as the ratio:

$$CS_n = 100\frac{NC_n - 1}{D - 1}$$

where NC_n is the number of *concepts in the layer n*. As we mention below, CS_n is an indication of the complexity of an RCM and complements the *SS* metric, since it is not the same a *SS* = 10% with a *CS* = 10% than with a *CS* = 90% (the latter is a symptom of the semantic fields containing many concepts). Note that both metrics make sense when the total number of definitions in the RCM is greater than 1.

As an example, let us evaluate these metrics for the RCM of model in Figure 2. The RCM collects 21 definitions. In the first layer, we can identify 10 primary concepts that we can group into 7 semantic fields: SF_{1a}= {Representation, Description}, SF_{1b}= {Specification}, SF_{1c}= {Simplification, Abstraction}, SF_{1d}= {Set}, SF_{1e}={Entity, Subject}, SF_{1f}={System}, SF_{1g}={Statement}. Therefore, SS_1 = 100*(7 – 1) / (21 – 1) = 30% and CS_1 = 45%, indicating that the RCM has a medium SS_1 and that the semantic fields have few concepts.

The global measure of scattering is defined as the average of each of these metrics considering all layers.

4.2. Density of references per concept and semantic field

As we have commented since the introduction, RCM contributes to facilitating the task of finding and sharing a common vocabulary for the development of any area of knowledge. An interesting exercise in this line is to find the most frequently used concepts in the relevant literature to define a term. For a particular RCM, these concepts can be determined by counting the number of references that traverse each concept, above all in the first layers of the RCM.

We can define the *density of references per concept in layer n* (DRC_n) as a concept metric calculated as the percentage obtained dividing the number of references that reach a concept in the layer *n* by the total of references of the RCM. Similarly, we define *density of references per semantic field in layer n* ($DRSF_n$). For example, the most cited primary concept in the RCM in Figure 2 is "*representation*", because it appears in 6 references out of a total of 21 references and therefore DRC_1(representation) = 28.6%.

A related metric would be determined calculating an aggregate of some quality indicator of the references that reach a specific concept or semantic field. For example we could add up the impact factor of the references of a concept and, thus, determine another quality metric of the concepts in an RCM.

4.3. Quality of definitions

Several authors have carried out various classifications of definitions and have analyzed the features that a good definition must meet, especially for *genus and differentia definitions* [13] and *lexical definitions* [14]. Taken these features as evaluation criteria, we propose the following metrics to assess of the quality of the definitions in an RCM.

The feature "*a definition must set out the essential attributes of the defined thing"* leads to an essentiality metric. The problem here is determining quantitatively which are the essential attributes for a certain definition. Consequently, what we propose is to associate an "essentiality" factor to each concept in a definition, and to determine the *definition essentiality* (*DE*) metric as the weighted mean of the essentiality of its concepts, weighted by the layer. The criterion for the determination of the essentiality of each concept is its *density of references* (see the previous section), in such a way that *DE* of a definition *d* is

$$DE(d) = \sum_{c\ in\ d} \frac{DRC_n(c) \cdot (N - n + 1)}{\sum_{i=1}^{N} i}$$

N being the number of layers of an RCM, and *n* the layer to which each *c* in *d* belongs. Let us study some examples from RCM of Model in Figure 2, which has three layers (N=3): the definition "model is a simplification of an original and it's for a purpose", has three concepts, being DRC_1(simplification) = 20%, DRC_2(original) = 25%, and DRC_3(purpose) = 35%, which results in a *DE* = 20*3/6 + 25*2/6 + 35*1/6 = 24.17%. Similarly, the definition "model is a subject (5%) with a membership-function (5%)" has a DE = 5*3/6 + 5*2/6 = 4.17%.

Another feature is *"definitions should avoid circularity"*: RCM can help us to detect circularities in a definition. To do this we should have the RCMs of the primary concepts of the definition and proceed to analyze them together. For example, in the RCM in Figure 2 we can see that "a model is a system ..." If, in another RCM system were defined as a model, we would have detected a circularity in the definitions of model and system.

The definition must not be too wide or too narrow: in an RCM the coexistence of general and specific concepts at the same layer is an indication of some definitions probably excluding individuals to which the definition applies or including others to which the definition is really not applicable. Apart from cases where the generality is obvious, such as in the definition "everything is a model" of the RCM of Figure 2, the generality and specificity of a definition are difficult to assess properties. However we could obtain a measure of the generality if we studied together a number of RCMs of a certain domain of knowledge. For example, we can determine that the simple definition "man is a living being" is general if we compare it with other RCMs of other living beings, for instance, cat, dog or tree. These RCMs will surely include this proposition among their definitions (but we all know that a cat, a dog or a tree are not men, despite the fact that they are living beings). On the contrary, "man is a living being with intelligence" is not general.

The definition must not be obscure: an analysis of the concepts of the definition can help to identify whether clear concepts are being used or not. This metric must be evaluated subjectively as the percentage of concepts considered obscure in a definition weighted by the layer.

A definition should not be negative where it can be positive: this feature can be mainly evaluated studying the linking phrases directly linked to the main concept, looking for negative forms of verbs.

4.4. Similarity in definitions

An RCM allows comparison of the main concept definitions layer by layer, highlighting those which are similar by considering the number of the concepts they share. This analysis allows us to obtain a measure of *similarity in definitions* (*SD*), a metric applicable to a definition *d*, calculated as the average of the percentages of similarity *SD(d, d')* of *d* with regard to the other definitions *d'*, being

$$SD(d, d') = 100 \frac{\text{number of common concepts between } d \text{ and } d'}{\text{maximum number of concepts in } d \text{ or } d'}$$

For example the definitions "*model* is a *statement* about a *system"* and "*model* is a *set* of *statements* about a *system"* has a high degree of similarity because the former share two concepts (statement and system) with the latter, which has three concepts, being therefore its *SD* factor 66% .We can obtain the average of the similarity between each definition and the other 20 definitions of model. Thus, the definition "Model is a representation of a system" is the definition with the maximum average similarity (29.33%) to the rest of definitions in the RCM of Model, whereas the definition "Model is a subject associated with a membership function" has an average similarity of 0% because it does not share any concept with the rest of definitions. For the computation of this metric we must exclude those definitions that use negative forms of verbs (e.g., "a table has legs" and "a table does not have legs" are not similar, though they share all their concepts).

4.5. Contradictions and inconsistencies

The synonymy between the concepts is not the only major element in the analysis of an RCM. The inspection and comparison of the concepts at a certain layer could lead to the detection of conflicting definitions (if they use opposing concepts). In this sense, the possible existence of linking phrases that negate a concept must be taken into account. Finally, inconsistencies can be detected when definitions lead to logical contradictions. We define the contradiction or inconsistency ratio (*CI*) of an RCM as the percentage of the number of definitions with contradictions and/or inconsistencies with regard to the total number of definitions. For example, a researcher on modelling theory could easily see that the definitions "model is a set of statements" and "model is a system" are contradictory, considering the classical definition of system [15] as an entity with related elements (the main difference is that the elements in a set are not necessarily interrelated, whereas the elements of a system are). In our example, we only consider this contradiction and therefore *CI* =100 x 1/21=4.76%.

4.6. Similarities between RCMs

Besides the similarities between definitions of the same main concept, it is interesting to look for similarities between the main concepts of different RCMs in order to detect synonyms, antonyms, and inconsistencies. Thus, for example, the definitions "alphabet is a set of symbols", "language is a system of symbols" and "language is a set of systems" enable the detection of possible similarities or inconsistencies in the RCMs of *alphabet* and *language*. The *similarity between RCMs* (*SRCM*) metric is calculated as the average of the similarity between the definitions of these RCMs (see the calculation of the similarity of a definition above).

4.7. Dependences between definitions

The fact that the references in a certain layer for a given concept are a subset of the references of another concept in the same layer is an indication of the existence of connections between definitions. For example, in the RCM of *model* all references using "specification" as a secondary concept also use "description" as a secondary concept (but not vice versa). This indicates that the authors consider that both aspects ("description" and "specification") are bound by some kind of relationship (indeed, in this case they are stating that there are two basic approaches to the *model* definition, which would be incomplete if only one of them was considered). This metric is also an indicator of the relevance of the definition. For example, the non-inclusion of the set of "description" references in the set of "specification" references indicates, moreover, that most authors consider "description" a more relevant concept in the definition.

4.8. Summary of metrics

Table 2 summarizes the above described metrics (first column), the scope, i.e. the aspect to which the metric refers (second column), and the criteria to be observed for the metric.

Table 2. Summary of metrics

Metric	Scope	Criteria
Concept and semantic scattering	Layer	Same semantic field
Density of references	Concept; Semantic field	Percentage of references
Quality	Definition	Relevance; Circularity; Generality and specificity; 'Darkness'; Non-denial
Similarity	Definitions	Average of percentage of common concepts in definitions
Contradictions and inconsistencies	Definitions	Detection of conflicting definitions; Logic contradiction
Similarities	RCMs	Synonymy; Antonym; Inconsistencies
Dependences	Definitions	Inclusion of concepts in references; Relevance of concepts

5. Related work

Graphical tools for knowledge representation are an effective aid in different fields such as artificial intelligence, learning support, semantic web, philosophy, etc. Many different proposals of graphical tools with practical applications exist. Our proposal initially arose from the need to represent knowledge related to the confusing terminology of modelling theory. We needed a tool that not only helped us to represent concisely the concepts contained in the definitions. It should also be useful for analyzing the origin of the definitions and for facilitating the understanding of their relationships.

Most existing concept mapping proposals can represent concepts or ideas and their relationships, but there are some differences, sometimes subtle, that led us to assess which among them might be useful in our context. Indeed, it is even quite common to confuse some of these proposal between each other since sometimes their diagrams look very similar (see for example [16,17] where the term "Concept Maps" is used to encompass other proposals of knowledge maps). Following, we survey the main options we evaluated.

MindMaps [18] are suitable for representing and informally outlining ideas, but we dismissed them because, due to being focused on ideas, they do not clearly show the importance of a concept in order to understanding the meaning of another [19]. Knowledge Maps [20], Cognitive Maps [21] and Argument Maps [22] are similar to MindMaps. They more formally define types of relationships between ideas and are aimed at improving learning and problem solving. We also dismissed them because they are based on the relationship between ideas rather than between concepts. In addition, Knowledge and Argument Maps require associating a type with the relationships between concepts that decreases the flexibility to represent the definitions found in the literature. Conceptual Graphs [23] is an interesting proposal that allows one to display relationships between concepts. However, these relationships are restricted according to rules related to logic and, therefore, we did not find them useful for representing more general concept definitions.

Another interesting group of solutions come from the field of ontology and semantics, standing out from others Topic Maps [24] and Annotation Ontologies [25,26]. Topic Maps is a standard for the representation of knowledge that offers many possibilities for structurally storing not only concepts ("Topics") and their relationships, but also associated resources, roles, alternative names, and so on. However, although Topic Maps can be represented graphically, they are more oriented to textual representation, using the XTM language. On the other hand, Annotation Ontologies are a formal solution to link ontology-based metadata to resources such as documents, but they are more oriented to Semantic Web representations, especially based on OWL/RDF (as we shall see below, there are mapping solutions between these solutions and Concept Maps that allow expressing RCMs as Topic Maps or in OWL/RDF). This would be an interesting complementary solution for an objective that were the reverse of ours, since, starting from a document corresponding to a bibliographic reference, it allows linking to it metadata such as the concepts appearing in the definitions included in that reference.

Finally, our focus was on Concept Maps, according to the formulation proposed by Novak [3,4]. This approach is based on the work of Ausubel about psychology of learning [27]. Ausubel claimed that "a meaningful learning only takes place when new concepts are connected to known concepts". We thought that this approach was applicable to our problem of understanding a definition from another existing one, visualizing what concepts they share and how they are related. Other interesting aspects of Concept Maps that led us to select it as our starting point were its orientation to concepts and relationships (rather than an orientation to ideas), its flexibility, its ease of use, and the fact of being one of the most popular solutions today. Concept Maps flexibility could be considered a drawback, since it could be a barrier to their formal study. However, as stated in [28] this apparent limitation does not preclude formal analysis using appropriate tools and utilities that facilitate the connection of Concept Maps with solutions close to artificial intelligence (ontologies, lexical databases, etc.). Likewise, there are proposals for the mapping between Concept Maps and some standards such as the abovementioned Topic Maps [29] or OWL/RDF [30]. This complementarity between various solutions to knowledge mapping could be extended even further, promoting the unification of some of these solutions using a common tool, as some authors suggest [19,31].

Therefore Concept Maps is a good starting point, but it does not fulfil all our requirements. In Concept Maps, propositions derived from the concepts and their relationships are based only on semantic units (node-link-node triads). This limitation is observed in [32], where it is stated that "sometimes (…) the order in which propositions are read is important", and it is proposed to improve Concept Maps by associating a number to each link to indicate the reading order. Even with this improvement, Concept Maps does not allow us to represent different propositions about the same set of concepts, since, as there is only one number per link, each triad could only be part of one proposition. RCM solves this problem differently, by providing Concept Maps with the possibility of representing different propositions using labelled paths, whereas the reading order is a consequence of the requirement of using unidirectional arrows. Moreover, in Concept Maps, the authorship of the propositions is not explicit (especially when they are created collaboratively) or simply refers to a single author (the creator of the concept map). Associating the directed labelled paths with bibliographic references, RCM introduces a new dimension in Concept Maps, because it allows us to know the author of a particular proposition (definition in RCM). As shown in this work, this allows us to perform interesting analysis focused on the concepts, the various propositions on a main concept (definitions) and the bibliographic references (authorship), as well as the relationships between them (which is a particular application of the cross-links proposed by Novak [3,4]).

Another aspect introduced by RCM is the intuitive characterization of the concepts relevance based on criteria established by the RCM author. We propose that the RCM author analyzes the significance of the concepts present in a definition and assigns a “layer” to each concept, encoding the layer by varying the intensity of a colour (the higher colour intensity, the greater importance). This way of working eases the identification of the concepts considered most relevant by the author, beyond its location in the hierarchy of the underlying concept map.

To the best of our knowledge, there is no proposal that uses Concept Maps to combine definitions and bibliographic references. There are separate solutions to both issues: Word Maps (also known as Concept Definition Maps) [33] is an interesting approach to teaching, especially to children, the meaning of words using diagrams; ConceptBib [34] proposes the application of Concept Maps to references; and finally, CiteWiz [35] uses an interactive concept map in order to showing keywords and co-authorship within scientific citation networks. All these approaches focus separately on definitions, references and citations without addressing their relationship.

Finally some words about tools for knowledge map design. As we have based RCM on Concept Maps, we have revised tools such as CMap Tools and VUE, which facilitate the construction of concept maps diagrams and offer additional features such as linking to resources, concept maps sharing, graphical styles configuration, etc. However, to our knowledge, there is no Concept Map tool that lets users select a *path* in order to label it with a reference, or to associate it with a resource (e.g. a link to the referenced document). This missing functionality forces RCM creators (1) to use linking phrases or concept boxes to contain the references, then misusing the linking phrases or concept boxes for a purpose that is not theirs; and (2) to repeat the process of reference association for every link phrase along the path. Similarly, the association of resources to the path should follow the same repetitive process. Moreover, Concept Maps tools generate the propositions (definitions in RCM) included in the concept map. However, in the case of RCM, this extraction of definitions is unsatisfactory for two reasons: (1) the extracted definitions lose the references where they were proposed, and (2), in the case of definitions included in other definitions, the tool does not extract the included definition, but only the longest one. All these facts show us that RCM proposes features that improve Concept Maps with new elements (labelled paths, concept layering) that are not present in common tools for concept maps.

To make the understanding and comparison between the different options easier, and to show in detail their similarities and differences with RCM, we present a summary table (Table 3) that collects some of the most relevant evaluated proposals. This table complements other comparative studies such as [19,29,31,34–38], comparing the characteristics we consider more relevant for our area of study. Due to the number of comparison characteristics collected, we have had to split the table into three fragments. We have added the bibliographic reference of each proposal. By doing so, we solve potential ambiguities, since there are different proposals that share the same name. It should also be noted that we have included the collected characteristics according to the original proposals of the authors, or occasionally, according to widely accepted subsequent improvements. You can find variations or tools that extend the functionality of a given solution by adding features offered by other solutions.

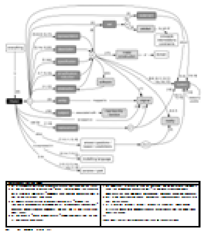
Table 3(a). Summary of concept mapping proposals

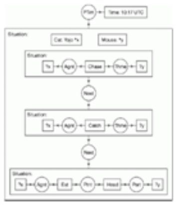
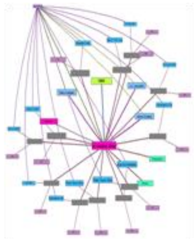
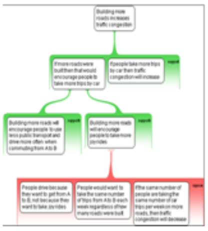
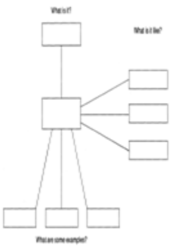
	Elements	Topology	Typical application	Focus	Several Focuses	Propositions
RCM	Main concept Concept layering Path labelling References	Mainly hierarchical	Analysis and comparison of definitions and bibliographic references	Main concept	No	Several paths through the same node
Concept Maps [3,4]	Concepts Linking phrases	Hierarchical Network	Knowledge representation Improve learning	Focus question	Allowed	Concept-Link-Concept triads
Mind maps [18]	Idea Association	Hierarchical	Brain-storming Memory retention	Central idea	No	Inside nodes
Conceptual graphs [23]	Concepts Instances Conceptual relations Situations	Network	Logical reasoning	Proposition	Allowed	Single
Topic maps [24]	Topics Associations Resources Types Names	Network	Knowledge representation	Not required	Allowed	No
Knowledge Maps [20]	Ideas Links	Network	Improve learning	Central idea	Allowed	Inside nodes
Argument Maps [22]	Claims Reasons Objections Rebuttals	Hierarchical	Reasoning learning	Conclusion	No	Path to conclusion
Annotation ontology [23,26]	Concepts Properties Relations	Network	Metadata sharing	Resource	Allowed	No
Word maps [33]	Main concept Category Properties Illustrations	Radial	Vocabulary learning	Main concept	No	No
Cognitive maps [21]	Ideas Links	Network	Problem solving	Focal point	Allowed	Inside nodes

Table 3(b). Summary of concept mapping proposals

	Typical reading	Labelled links	Directional links	Cross links	Typed links	Typed nodes	Graphical formats	Standard
RCM	Left to right	Required	Unidirectional	Allowed	No	No	Colour-coded	No
Concept Maps	Top to bottom	Allowed	Required	Allowed	No	No	Allowed	No
Mind maps	Radial	No	Required	No	No	No	Allowed	No
Conceptual graphs	Any	Required	Required	No	Required	Allowed	No	ISO/IEC 24707
Topic maps	Any	Required	Allowed	Allowed	Allowed	Yes	No	ISO/IEC 13250
Knowledge Maps	Any	Required	Required	No	Required	No	No	No
Argument Maps	Bottom to Top	Allowed	Allowed	No	Required	No	Allowed	No
Annotation ontology	Radial	Required	Required	No	Required	Allowed	No	No
Word maps	Radial	No	No	No	No	No	No	No
Cognitive maps		Only "-" label	Required	No	Allowed	No	Allowed	No

Table 3(c). Summary of concept mapping proposals

	Typical serialization formats	Links to resources	Roles of nodes	Software	Graphical format	Sample thumbnail	Thumbnail source
RCM	CXL, XTM, XCM, IVML	Allowed	No	Under development	Required		This paper
Concept Maps	CXL, XTM, XCM, IVML	Allowed	No	Cmap Tools VUE	Required		[4]
Mind maps	No	Allowed	No	Free Mind Xmind iMindMap	Required		[36]

	Typical serialization formats	Links to resources	Roles of nodes	Software	Graphical format	Sample thumbnail	Thumbnail source
Conceptual graphs	CGIF	No	No	CharGer Amine	Required		[23]
Topic maps	XTM	Allowed	Allowed	Topic Map Designer Wandora DigiDocMap	Allowed		Wandora, http://www.wandora.org/wandora/wiki/index.php?title=Screenshots
Knowledge Maps	No	No	No	KP-Lab Tools	Required		[20]
Argument Maps	AIF LKIF	No	No	Araucaria Carneades Rationale iLogos	Required		Wikimedia, http://commons.wikimedia.org/wiki/File:Traffic_congestion_straw_man.png
Annotation ontology	RDF	Allowed	No	OBOEdit	Allowed		[25]
Word maps	No	No	No	General Graphic Tools	Required		[33]

	Typical serialization formats	Links to resources	Roles of nodes	Software	Graphical format	Sample thumbnail	Thumbnail source
Cognitive maps	No	No	No	Decision Explorer	Required		[21]

6. Conclusions and future work

In this paper we have described a tool, called References-enriched Concept Maps, whose main aim is to facilitate the analysis of the different definitions regarding the same term that exist in the relevant literature. RCMs allow one to obtain a "picture" that reflects the state of the art in the discussion on the meanings of a given concept. Furthermore, RCMs complement that picture with references to the literature sources that allow the user to retrieve the definitions as close as possible to the way in which they were originally presented.

Although the process of creation of an RCM is relatively simple, its application can lead to very complex diagrams depending on the complexity of the analyzed concept: a medium sized RCM smaller than half a page, can contain tens of definitions. It is important to note that an RCM never decreases the inherent complexity of the defined concept, but rather it helps to address this complexity and, more importantly, its analysis helps one to understand the different points of view of the authors who have investigated its meaning. By doing so, RCMs contribute to finding answers to different questions that arise in literature concerning controversial concepts.

RCMs also constitute an excellent tool as motivation for raising new questions about the definitions they cover. An overview of all the definitions, together with the analytical data described in the paper, enables one to find emerging issues related to the scope of the main concept. For example, an RCM user, in light of the performed analysis, can obtain new and more complete definitions from the relevant definitions present in the RCM.

It is worth noting that in this paper we have focused on the application of RCM to compare definitions. However, researches can give RCM other uses, such as the description of features of a certain subject, as in the work of [39]. We emphasize that an RCM is a concept map enriched with a path labelling, which complements the knowledge gathered on the map with references to the authors of this knowledge. Therefore, an RCM can be used in any area where concept maps also apply.

Tools for creating Concept Maps can also be used for the creation of RCMs. However, the particularities presented in this paper that differentiate RCMs from Concept Maps advise developing new tools to facilitate the creation of the RCMs. These tools should take into account all the unique features of RCMs, such as path labelling with references, creation of list of references, concept layers, etc. Furthermore, automation in order to obtain quantifiable analysis metrics is another area for future work. For the metrics whose criteria are related to aspects such as synonyms or antonyms, it is interesting to research its integration with tools such as Wordnet [40] in order to facilitate the automation of the computation of such indicators.

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