New probabilistic price forecasting models: Application to the Iberian Electricity Market

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Abstract

This article presents original Probabilistic Price Forecasting Models, for day-ahead hourly price forecasts in electricity markets, based on a Nadaraya–Watson Kernel Density Estimator approach. A Gaussian Kernel Density Estimator function is used for each input variable, which allows to calculate the parameters of the probability density function (PDF) of a Beta distribution for the hourly price variable. Thus, valuable information is obtained from PDFs such as point forecasts, variance values, quantiles, probabilities of prices, and time series representations of forecast uncertainty. A Reliability Indicator is also introduced to give a measure of “reliability” of forecasts. The Probabilistic Price Forecasting Models were satisfactorily applied to the real-world case study of the Iberian Electricity Market. Input variables of these models include recent prices, power demands and power generations in the previous day, power demands in the previous week, forecasts of demand, wind power generation and weather for the day-ahead, and chronological data. The best model, corresponding to the best combination of input variables that achieves the lowest MAE, obtains one of the highest Reliability Indicator values. A systematic analysis of MAE values of the Probabilistic Price Forecasting Models for different combinations of input variables showed that as more types of input variables were considered in these models, MAE values improved and Reliability Indicator values usually increased.

Keywords: short-term forecasting; market prices; Iberian electricity market; electricity prices; probabilistic forecast

Nomenclature

Abbreviations

PPFM	Probabilistic Price Forecasting Model
DAEPF	Day-Ahead Electricity Price Forecasting
NW-KDE	Nadaraya–Watson Kernel Density Estimator
PDF	probability density function
MAE	Mean Average Error
RI	Reliability Indicator
SVM	Support Vector Machine
MIBEL	Iberian Electricity Market
OMIE	Market Operator of the MIBEL
TSO	Transmission System Operator
REE	Spanish Transmission System Operator
REN	Portuguese Transmission System Operator
NWP	Numerical Weather Prediction
GFS	Global Forecast System

Variables

x	explanatory price variable
y	electricity price variable
$x_{v,new}$	explanatory price variable for dimension v for future hour (*new*)
$x_{v,p}$	explanatory price variable for dimension v for past instant p
$\{x_{1,new}, \ldots, x_{v,new}, \ldots, x_{m,new}\}$	generic case for future hour (*new*)
x_{new}	explanatory price variable for future hour (*new*)
x_v	explanatory price variable for dimension v
y_{new}	electricity price variable for future hour (*new*)

y'_{new} standardized electricity price variable between 0 and 1 corresponding to y_{new}
y_p electricity price variable for instant p
y'_p standardized variable between 0 and 1 corresponding to y_p

Elements

$[Y_{new}]$ element associated with y_{new}
$[Y'_{new}]$ element associated with y'_{new}
$[X_{new}]$ set of elements associated with explanatory variables $x_{v,new}$.

Numerical values

n number of cases of the historical dataset
m number of price explanatory variables
$x_{new,max}$ highest limit value for x_{new}
$x_{new,min}$ lowest limit value for x_{new}
x_{v_max} maximum value of x_v of the historical dataset of cases
x_{v_min} minimum value of x_v of the historical dataset of cases
$y_{new,max}$ the highest limit value for y_{new}
$y_{new,min}$ the lowest limit value for y_{new}
$n\text{-}new,min$ number related to $y_{new,min}$
$n\text{-}new,max$ number related to $y_{new,max}$
bl_a basic level of activation
bl_{a_std} standardized basic level of activation
F_{chg} factor between 0 and 1
N_p minimum number of activated points from the historical dataset
NQ total number intervals
N number of elements in the out-sample dataset

Parameters

h_v bandwidth for dimension v
α_{new} parameter α of PDF of a Beta distribution for future hour (*new*)
β_{new} parameter β of PDF of a Beta distribution for future hour (*new*)
$(\alpha_{new}, \beta_{new}, y_{new,min}, y_{new,max})$ parameters of a PDF of a Beta distribution for future hour (*new*)
$(\alpha_{new\_q}, \beta_{new\_q}, y_{new\_q,min}, y_{new\_q,max})$ parameters of a PDF of a Beta distribution for new case q

Functions

$K_v(x_{v,p}; x_{v,new}, h_v)$ kernel density estimation function
$F_{new}(\alpha_{new}, \beta_{new}, y_{new,min}, y_{new,max})$ Beta cumulative distribution function for future hour (*new*)
$f(y'_{new}; \alpha_{new}, \beta_{new})$ PDF of a Beta distribution, in [0 ; 1], for y'_{new}
$f_{new}(y_{new}; \alpha_{new}, \beta_{new}, y_{new,min}; y_{new,max})$ PDF of a Beta distribution for future hour (*new*)
$F_{new\_q}(y_q; \alpha_{new\_q}, \beta_{new\_q}, y_{new\_q,min}, y_{new\_q,max})$ Beta cumulative distribution function for new case q for future hour (*new*)

Frequencies

$f_{obs,i}$ observed frequencies in the interval i
$f_{tar,i}$ target frequency in the interval i

Data for application of PPFM models to MIBEL

$p_{D\text{-}6,h}$ price for hour h of day D-6
$p_{D,h}$ price for hour h of the day D
w_{D+1} week
$\hat{T}_{D+1,h|D,t}$ regional weighted forecasted hourly temperature for hour h of day D+1, obtained at hour t of the day D
$\hat{I}_{D+1,h|D,t}$ regional weighted forecasted hourly irradiations for hour h of day D+1, obtained at hour t of the day D
$\hat{v}_{D+1,h|D,t}$ regional weighted forecasted hourly wind speeds for hour h of day D+1, obtained at hour t of the day D
$HG_{D\text{-}1,h}$ hydropower generation for hour h of day D-1
$SG_{D\text{-}1,h}$ solar power generation and power cogeneration for hour h of day D-1
$CG_{D\text{-}1,h}$ coal power generation for hour h of day D-1
$CCG_{D\text{-}1,h}$ combined cycle power generation for hour h of day D-1
$WG_{D\text{-}1,h}$ wind power generation hour h of day D-1
$NG_{D\text{-}1,h}$ nuclear power generation hour h of day D-1
$LD_{D\text{-}1,h}$ power demand hour h of day D-1
$LD_{D\text{-}6,h}$ power demand hour h of day D-6
$\hat{WG}_{D+1,h|D,t}$ wind power generation forecast for hour h of day D+1, obtained at hour t of the day D
$\hat{LD}_{D+1,h|D,t}$ power demand forecast for hour h of day D+1, obtained at hour t of the day D
P_{real_T} real hourly price value for the hour T

Forecasts from the application of PPFM models to MIBEL

$\hat{p}^{d}_{D+1,h\|D,t}$	day-ahead hourly price forecast for hour h of day D+1, obtained at hour t of the day D
$P_{forecast\_T}$	expected value of hourly price of a PPMF model for the hour T.

1. Introduction

1.1. *Context of this research related to the day-ahead electricity price forecasting*

In the last 25 years the transition from monopolistic power sectors to competitive ones has led to the trade of electricity under market rules. In the market paradigm, the economic operation of the power system is based on price signals that are holistically influenced by all the market agents, which try to achieve their economic interests. These price signals are mainly generated in the day-ahead market, which influences all the remaining market prices (e.g., intraday markets, derivative product markets, and bilateral markets). Knowing in advance the prices that will be settled by the day-ahead electricity market is essential for the price-makers' agents, guiding the bidding decision-making process in order to maximize their economic profit.

Furthermore, knowing the prices in advance is even important for the least relevant agents (those who, due to their low volume of trading operations, do not influence the price value), who can evaluate the risk of anomalous prices in advance and, consequently, respond to high or low price forecasts by managing their electricity consumption and/or their self-power generation.

Therefore, a significant effort has been made in recent years to develop Day-Ahead Electricity Price Forecasting models (DAEPF models). Most of the published DAEPF models are focused on point (spot) forecasts, that is, they provide the values of forecasted hourly electricity prices without any additional information. However, these DAEPF models that only give point forecasts can be inadequate for trading purposes because they do not show the uncertainty associated with the price forecasts, which is essential for risk-based market decisions. Point (spot) forecasts do not offer information to support trading associated with market risks. Market decisions based on point forecasts are suitable when the economic impact of the deviation (between forecast value and real value) is linearly dependent on the absolute value of the deviation. However, probabilistic price forecasting models are able to provide information regarding the uncertainty of the price forecasts. They have become essential models for making proper risk-based decisions when the magnitude of the price (or magnitude of the deviation) has a higher relative impact on more extreme price values, that is, when it is imperative to know the probability of occurrence of the different level of prices (or magnitude of deviations).

1.2. Literature review

DAEPF models centered on point (spot) forecasts are essentially based on classic time series models or computational intelligence models, although some models combine both approaches. Classic time series DAEPF models described in the literature use a wide variety of techniques [1]. Some works describe the application of only one technique [2], while others apply a set of techniques for developing several models and comparing their forecasting results. The techniques used include exponential smoothing [3], multiple regression [4, 5], time-varying regression [6, 7], Box-Jenkins and derived models [8-10], econometric models [11] and GARCH (Generalized Auto-Regressive Conditional Heteroskedasticity) [12, 13]. Usually, the simpler techniques are used to develop benchmark models for comparison purposes [14]. Computational intelligence DAEPF models are based principally on artificial neural networks [15-17], fuzzy systems [18, 19], and support vector machines (SVM) [20, 21]. Some of both families of DAEPF models need explanatory variables such as load demand, meteorological variables (mainly temperature), and wind or solar electric power production, the values of which, for the forecasting horizon, are obtained by means of other short-term forecasting models [22, 23]. Preprocessing techniques such as wavelet decomposition have been used to improve model performance [24], optimization methods have been used to tune the structure of the DAEPF model [25-27], or classifiers have been used as an additional module to forecast price spikes [28]. Combinations of forecasts from different DAEPF models have been studied, and reveal a more accurate and robust point forecast [5, 29]. Similar approaches, using advanced forecasting techniques, have been also applied in other electric energy sectors, such as load forecasting [30], wind power forecasting [31], solar power forecasting [32], or building energy demand forecasting [33].

Electricity market agents can need price forecasts for risk-based decisions in diverse situations. They can use probabilistic price forecasts for risk-based decisions by pondering the risk based on probability measures. The impact of these decisions can depend on the probability of having prices higher/lower than a threshold value. In this sense, probabilistic price forecasting models have an important added value compared with non-probabilistic price forecasting models. Probabilistic models also overcome the limitations of spot forecasts by allowing the generation of scenarios for the day-ahead for several kinds of power-producers [34-38]. Some kinds of DAEPF probabilistic models have been identified in the literature [1] according to their output values: interval forecasts (or also known as prediction intervals), density forecasts, and threshold forecasts, although this last kind can be considered as a special instance of prediction intervals.

Most of the early works describing DAEPF probabilistic models focused on prediction intervals, since estimating these can help utilities and independent power producers to submit effective bids with low risks [38]. The techniques utilized include support vector machines with heteroscedastic variance equation and the maximum likelihood estimation [39], an extreme learning machine with bootstrapping method [23] or with an estimation of the noise variance [40], delta and bootstrap methods [41], a recursive dynamic factor analysis [42], the variational heteroscedastic Gaussian process [43], and quantile regression averaging with principal component analysis [44]. The main application of threshold forecasting is related to demand-side management, where several data-mining techniques used as classifiers have been used [45, 46]. In spite of everything, interval and threshold forecasts only allow decisions related to the corresponding specific intervals or thresholds. However, density forecast, which is the subject of the probabilistic price forecasting models presented in this article, provides suitable probabilistic information for any risk-based decision usage.

A much better approach to building DAEPF probabilistic models is the forecasting of the probability density function (PDF) or density forecasting. In any field of application, a density forecast of a random variable at some future time is an estimate of the probability distribution of the possible future values of that variable. On one hand, it gives a complete description of the uncertainty associated with a prediction and overcomes the main deficiency of a point forecast (from non-probabilistic models), which does not provide any information about the associated uncertainty [47]. On the other hand, obtaining an entire forecast density provides more complete information than single prediction intervals. However, there are only a few works that deal with the development of short-term probabilistic forecasting models based on PDFs for electric energy applications, such as load forecasting [48, 49], solar power forecasting [50, 51], wind power forecasting [52, 53], or electricity price forecasting [54, 55].

Thus, the most interesting DAEPF probabilistic models (i.e., probabilistic price forecasting models for probabilistic forecasts of the day-ahead hourly price) are focused on probability density functions (PDFs); however there is a scarcity of probabilistic price forecasting models based on them. In the case of electricity price density forecast, Serinaldi [54] proposes the use of the Generalized Additive Models for Location, Scale, and Shape (GAMLSS) [56] to obtain the entire day-ahead electricity price distribution function; the explanatory variables used by the forecasting model are past prices, electricity loads (including the day-ahead forecasts, if available) and chronological information. Another approach is the one proposed by Jónsson *et al.* [55], where the densities for the day-ahead electricity prices are obtained using a time-adaptive quantile regression model and the approximation of the distribution tails. The forecasting model uses as input variables forecasts of spot electricity prices (provided by a spot DAEPF model), forecasts of system load and forecasts of wind power production.

1.3. Objectives and characteristics of the models of this research and their application

This article describes original Probabilistic Price Forecasting Models (PPFM models) for probabilistic forecasts of the day-ahead hourly price in electricity markets focused on probability density functions (PDFs) of Beta distributions. Main novel characteristics of PPFM models and their application are outlined in the following paragraphs.

The PPFM models provide parametric probabilistic forecasts, represented by the PDF of a Beta distribution, for each hour of the forecast price (output variable). This parametric approach of the output variable provides significant advantages because it gives full information about the uncertainty of price forecasts useful for downstream risk-based decision-making tools.

The extensive set of input variables of the PPFM models consists of large historic time series of hourly prices in previous days, regional-aggregated hourly power generations (hydropower generation, wind power generation, solar power generation and power cogeneration, nuclear power generation, combined cycle power generation and coal power generation), and hourly power demand in the previous day and in the previous week. Furthermore, this set of input variables also includes hourly time series records of forecasts of regional-aggregated hourly power demands, forecasts of wind power generation and forecasts of weather (hourly wind speed, temperature, and irradiation) in the region for the day-ahead as well as chronological data. The intrinsic uncertainties associated with price forecasts are influenced by these different kinds of explanatory price variables (input variables). These variables affect the expected value of price, as well as the price forecast uncertainty.

The new PPFM models are based on a Nadaraya–Watson Kernel Density Estimator (NW-KDE) approach [57, 58] using a suitable Gaussian KDE function for each one of its input variables, i.e. explanatory price variables. The NW-KDE approach applied to historical cases (historical data of explanatory price variables and the hourly price historical data), directly obtains the corresponding expected and variance values. They allow us to calculate, in a straightforward way, the essential parameters of the probability density function (PDF) of a Beta distribution that describes the behavior of the price variable for each forecasting hour. Then, this PDF function allows the expected value of the hourly price variable to be determined (i.e. point forecast of hourly price) as well as a variety of useful quantitative probabilistic results of the probabilistic forecast of each PPFM model. These parametric representations (of PDFs) by Beta distributions are advantageous (for downstream usage of the forecast uncertainty) when compared with other approaches.

A new Reliability Indicator (*RI*) is introduced to assess the uncertainty associated with probabilistic price forecasts; the *RI* gives a quantitative measure (from 0% to 100%) of “reliability” of forecasts of PPFM models. This article also presents a novel representation of “reliability diagram”, associated with the *RI*, which allows the evaluation of the performance of the PDF of Beta distributions to be carried out. “Forecast cases” that fall outside the limits of the PDF are identified by such suitable “reliability diagram”.

The *RI* values for PPFM models in the real-life case study are also included in this article. Then, a comparison of the PPFM model´s performances revealed that *RI* (“reliability”) values usually improved, and that MAE error values improved when more input variable types (chronological variables, price variables, power demand variables, weather forecast variables, and power generation variables) were considered in the PPMF models.

An original procedure is used to calibrate the bandwidths of the kernel Gaussian functions involved in the NW-KDE approach. This procedure uses an iterative process to seek successive bandwidth values, for each explanatory variable, which in turn achieve a near optimal *RI* value for each “forecast case” (“new case”).

The real-life case study of the Iberian Electricity Market (MIBEL) was used to satisfactorily test the PPFM models. As far as we know, this is the first time that probabilistic price forecasting models, based on the NW-KDE approach, have successfully been applied to the Iberian Electricity Market. In this article, firstly, suitable studies of input variable selection for PPFM models led to the determination of reasonable combinations of variables (grouped by their common characteristics). The selection process allowed us to carry out a systematic analysis of Mean Absolute Error (MAE) values of PPFM models in the real-life case study, in order to find the best input variable combination associated with the lowest MAE value, that is, to find the best PPFM model. Furthermore, these studies could determine the relative importance of each input variable. Secondly, a description of the main results of forecast examples of this best PPFM model is included in terms of hourly probability density functions of Beta distributions for hourly price forecasts, time series representations of price forecast uncertainty, expected and variance values of hourly price, and other valuable probabilistic information. Examples of other valuable information are quantiles and probabilities relative to given values of the price (probabilities of prices exceeding or failing to exceed a threshold value). This information can be used to assess the risk in trading operations in electricity markets.

1.4. Contributions of this research

From the above sub-sections in this Introduction, the novelty and contributions of this article are summarized as follows:

- New PPFM models based on NW-KDE using Gaussian KDE functions for their input price explanatory variables.
- Very extensive set of input variables of PPFM models consisting of large time series of hourly historical values of price, power generations, demands, and forecasted values of demands, wind speeds, temperatures, and irradiations, as well as chronological data.
- Probability density functions of Beta distributions for the hourly prices of each PPFM model.
- Reliability Indicator (*RI*) to assess the uncertainty associated with probabilistic price forecasts of each PPFM model.
- Calibration procedure of bandwidths of the Gaussian functions of PPFM models.
- Straightforward determination of expected values of hourly prices, variance values, quantiles, probabilities of prices exceeding or failing to exceed a threshold value, and time series representations of forecast uncertainty of each PPFM model.
- Successful application of the new PPFM models to the Iberian Electricity Market (MIBEL). The first time that probabilistic price forecasting models based on NW-KDE are applied to MIBEL.

The PPFM models, their PDF functions for hourly price forecasts and practical probabilistic information from such PDF functions can be useful for agents of the day-ahead electricity markets and the power industry.

1.4. Structure of this article

The structure of this article is as follows: Section 2 describes PPFM models for probabilistic price forecasts and the methodology, mainly in terms of the NW-KDE approach. Section 3 contains a description of the time framework and data for the PPFM models in the real-life case study. The application of such PPFM models and the corresponding results are presented in Section 4. After the results, Section 5 proceeds to discuss them and depicts comparisons of performances of PPFM models from the point of view of MAE values and Reliability Indicator values. Finally, the conclusions of this article are presented in Section 6.

2. Probabilistic price forecasting models (PPFM models)

The probabilistic forecasting models (PPFM models) presented in this article are based on the Nadaraya–Watson Kernel Density Estimator (NW-KDE), the fundamentals of which were proposed by Nadaraya and Watson [57, 58]. A KDE approach is a non-parametric approach to estimate the PDF of a random variable. KDE methods have been applied satisfactorily to obtain wind and solar power probabilistic forecasts [59-61].

The NW-KDE mathematical method used in our work is more complex than other KDE approaches, with the main difference being that it allows the parameter values of the PDF of a Beta distribution for the output variable (hourly price variable) to be obtained directly from the expected and variance values, calculated basically by the applying the NW-KDE approach to the historic data of explanatory price variables and the historic data of hourly prices. In this way, such PDF functions allow useful probabilistic quantitative information of the price variable behavior for each hour of the forecasts to be determined.

The NW-KDE method was implemented in a novel NW-KDE probabilistic forecasting tool developed specifically for this research work, which also considered diverse advanced features. However, only essential aspects of the NW-KDE method used in the PPFM models are presented in this article due to reasons of simplicity.

In this Section 2, a mathematical description of PPFM models is presented; later the kernel density estimation function utilized in these models is explained; and, lastly, parameters related to probabilistic price forecasting models are described.

2.1. Mathematical description of probabilistic price forecasting models

Probabilistic price forecasting models consider a historical dataset composed of n cases, corresponding to past instants p (p = 1, 2, ... n), of m price explanatory variables, x, and the corresponding dependent variable (hourly electricity price variable, y). This set of n historical cases, composed by hourly time series of the mentioned variables, constitutes the knowledge base dataset (matrix of knowledge) represented in (1).

$$\begin{Bmatrix} x_{1,1} \ldots x_{v,1} \ldots\ x_{m,1} & y_1 \\ \vdots \quad \ddots \qquad\qquad \vdots & \vdots \\ x_{1,p} \quad x_{v,p} \quad x_{m,p} & y_p \\ \vdots \qquad\qquad \ddots \quad \vdots & \vdots \\ x_{1,n} \cdots x_{v,n} \cdots\ x_{m,n} & y_n \end{Bmatrix} \tag{1}$$

The PPFM models assume that the neighborhood of a future generic case $\{x_{1,new}, \ldots, x_{v,new}, \ldots, x_{m,new}\}$ for a future hour (*new*) can be assessed for each explanatory variable (dimension) v, by a kernel density estimation function $K_v(x_{v,p}; x_{v,new}, h_v)$, where $x_{v,new}$ plays the role of the parameter of the center of the kernel function and h_v corresponds to its bandwidth, that is, the width of activation of the kernel function in the neighborhood of the *new* case for the variable v. The kernel function $K_v(x_{v,p}; x_{v,new}, h_v)$ is a grade function, used to "select" mainly historical cases with values $x_{v,p}$ relatively "close" to each specified value of the variable $x_{v,new}$, by "activation values" given by such kernel function. The "join activation" for all dimensions, that is, for all explanatory variables, corresponds to the product of the kernel functions in all dimensions v, computed by the expression $\prod_{v=1}^{m} K_v\left(x_{v,p}; x_{v,new}, h_v\right)$, that is, a product kernel [62, 63].

In order to gain a better understanding of the mathematical description of the PPFM models, an illustrative example is included in Fig. 1. Let us consider, for simplicity, that there is only one price explanatory variable (v=1), the power demand variable. In Fig. 1 the price explanatory variable has a value of 40.5 GWh for a future case (value x_{new}). A suitable kernel density estimation function (for example, the one shown in the figure) will be used to assess the neighborhood of x_{new} considering the historic dataset. As shown in Fig. 1, for such value of x_{new}, a limit value $x_{new,max}$ of 42.5 GWh and a limit value $x_{new,min}$ of 38 GWh can be adequately determined (by using an appropriate basic level of activation of the kernel density estimation function), for purposes of practical computations to be carried out later. These limits of the explanatory variable values lead to determine limits of the price values between $y_{new,max}$ (32.2 €/MWh) and $y_{new,min}$ (16.0 €/MWh).

Coming back to the PPFM models, for the general situation of m explanatory variables and a *new* generic case $\{x_{1,new}, \ldots, x_{v,new}, \ldots, x_{m,new}\}$ for a future instant, let us consider that the element $\left[Y_{new}\right]$, associated to the variable y_{new} (hourly price variable) in the interval $[y_{new,min}\,; y_{new,max}]$, is converted to the element $\left[Y'_{new}\right]$ referred to a variable y'_{new} between 0 and 1 by a regular min-max standardization.

The expected value of $\left[Y'_{new}\right]$, for a *new* (future) instant, can be estimated by the weighted average of all values y'_p between 0 and 1 (corresponding to historical values y_p belonging to the interval $[y_{new,min}\,; y_{new,max}]$), where the weights are the kernel activation values ("join activation" values).

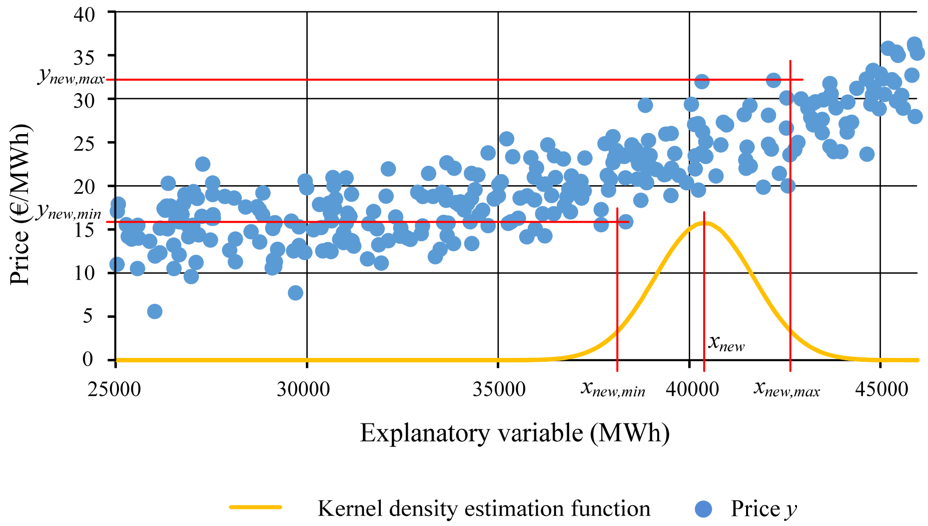

Fig. 1. Illustrative example.

Let us consider that the element $\left[X_{new}\right]$ is associated to v price explanatory variables $x_{v,new}$. The expected value of $\left[Y'_{new}\right]$ is the conditional expected value $E\left[Y'_{new}\middle|X_{new}\right]$ that can be estimated by (2), [63],

$$E\left[Y'_{new}\middle|X_{new}\right]=\frac{\sum_{p=n-new,min}^{n-new,max}\left(y'_p\cdot\prod_{v=1}^{m}K_v\left(x_{v,p};x_{v,new},h_v\right)\right)}{\sum_{p=n-new,min}^{n-new,max}\left(\prod_{v=1}^{m}K_v\left(x_{v,p};x_{v,new},h_v\right)\right)} \qquad (2)$$

where the number $n\text{-}new,min$ is related to $y_{new,min}$ and the number $n\text{-}new,max$ is related to $y_{new,max}$.

The approach used to represent $\left[Y'_{new}\right]$, standardized in the interval [0,1], allows us to obtain estimation values of the parameters α_{new} and β_{new} of the corresponding probability density function of a Beta distribution, as explained later, by using the value of $E\left[Y'_{new}\middle|X_{new}\right]$ according to (2), and the variance $V\left[Y'_{new}\middle|X_{new}\right]$ according to (3), [64].

$$V\left[Y'_{new}\middle|X_{new}\right]=E\left[\left(\left[Y'_{new}\right]\right)^2\middle|X_{new}\right]-\left(E\left[Y'_{new}\middle|X_{new}\right]\right)^2 \qquad (3)$$

Thus, the application of the NW-KDE method to the expected value of the square of the dependent variable $E\left[\left(\left[Y'_{new}\right]\right)^2\middle|X_{new}\right]$ and to the square of the expected value $\left(E\left[Y'_{new}\middle|X_{new}\right]\right)^2$, leads to obtain the variance $V\left[Y'_{new}\middle|X_{new}\right]$ that can be estimated by (4).

$$V\left[Y'_{new}\middle|X_{new}\right]=\frac{\sum_{p=n-new,min}^{n-new,max}\left(\left(y'_p\right)^2\cdot\prod_{v=1}^{m}K_v\left(x_{v,p};x_{v,new},h_v\right)\right)}{\sum_{p=n-new,min}^{n-new,max}\left(\prod_{v=1}^{m}K_v\left(x_{v,p};x_{v,new},h_v\right)\right)}-\left(\frac{\sum_{p=n-new,min}^{n-new,max}\left(y'_p\cdot\prod_{v=1}^{m}K_v\left(x_{v,p};x_{v,new},h_v\right)\right)}{\sum_{p=n-new,min}^{n-new,max}\left(\prod_{v=1}^{m}K_v\left(x_{v,p};x_{v,new},h_v\right)\right)}\right)^2 \quad (4)$$

The probability density function $f(y'_{new};\alpha_{new},\beta_{new})$ of a Beta distribution supported in the interval [0,1], for y'_{new} in the interval [0,1], is given by (5), [65, 66],

$$f(y'_{new};\alpha_{new},\beta_{new})=\frac{1}{B(\alpha_{new},\beta_{new})}y'^{\,\alpha_{new}-1}_{new}\cdot(1-y'_{new})^{\beta_{new}-1} \quad (5)$$

where $B(\alpha_{new}, \beta_{new})$ is a known normalization "constant" (if fixed values of α_{new}, β_{new}) [65], and the estimated values of the parameters α_{new} and β_{new}, corresponding to the *new* instant, are obtained from (6) and (7) using the method of moments [65].

$$\alpha_{new}=\frac{\left(1-E\left[Y'_{new}\middle|X_{new}\right]\right)\cdot\left(E\left[Y'_{new}\middle|X_{new}\right]\right)^2}{V\left[Y'_{new}\middle|X_{new}\right]}-E\left[Y'_{new}\middle|X_{new}\right] \quad (6)$$

$$\beta_{new}=\alpha_{new}\cdot\frac{\left(1-E\left[Y'_{new}\middle|X_{new}\right]\right)}{E\left[Y'_{new}\middle|X_{new}\right]} \quad (7)$$

The expected value of the hourly price, i.e., the point forecast of hourly price value, with prices values expressed in €/MWh in the interval [$y_{new,min}$; $y_{new,max}$], can be obtained by using a probability density function $f_{new}(y_{new}$; α_{new}, β_{new}, $y_{new,min}$; $y_{new,max})$ of a Beta distribution, supported in such interval [$y_{new,min}$; $y_{new,max}$], where y_{new} represents the variable associated to the hourly price. Thus the expected value of the hourly price is given by (8), [65, 66].

$$y_{new,min}+\left(y_{new,max}-y_{new,min}\right)\frac{\alpha_{new}}{\alpha_{new}+\beta_{new}} \quad (8)$$

Lastly, a variety of useful quantitative probabilistic information can be also obtained from the mentioned probability density function for the price forecast, as presented in Section 4.2.

2.2. Kernel density estimation function

Different kinds of kernel functions can be used. The Gaussian function is the most common and practical function to be applied to each dimension (each explanatory variable) *v* by (9).

$$K_v\left(x_{v,p};x_{v,new},h_v\right)=\frac{1}{h_v\sqrt{2\pi}}e^{-\frac{\left(x_{v,p}-x_{v,new}\right)^2}{2\cdot h_v^{\ 2}}} \quad (9)$$

Notice that $x_{v,p}$ plays the role of the independent variable in the kernel Gaussian function K_v ($x_{v,p}$; $x_{v,new}$, h_v) and $x_{v,new}$ and h_v acts as parameters. There is one kernel function for each variable v, and it is applied to each value $x_{v,p}$ of the historical cases.

Observe that $x_{v,new}$ is set as the center of the Gaussian function that is used to assess the neighborhood of $x_{v,new}$ (i.e., mainly historical cases p with values $x_{v,p}$ relatively "close" to $x_{v,new}$). Furthermore h_v is the Gaussian RMS width, or the Gaussian standard deviation parameter, and it represents the bandwidth of the kernel.

2.3. Parameters related to probabilistic price forecasting models

As mentioned at the beginning of this Section 2, the NW-KDE method was implemented in a novel NW-KDE probabilistic forecasting tool, which contains diverse parameters. Some of these parameters have to be calibrated in order to achieve an adequate performance of the PPFM models.

First, descriptions of parameters are provided (Section 2.3.1); and afterwards, a summary of a complex procedure of bandwidths calibration is briefly outlined to obtain satisfactory values of bandwidths (Section 2.3.2).

2.3.1. Descriptions of parameters

When working with a kernel estimator it is necessary to select the kernel function K_v and the smooth parameter or bandwidth h_v, for each explanatory variable (dimension v). The selection of K_v is a relatively less important problem, since different kernel functions that produce good results, can be utilized. In this article, the Gaussian kernel density function is applied as previously indicated in Section 2.2. An inconvenience of dealing with the Gaussian kernel density function is that if it is not limited, then it requires intensive computation for the historical dataset of cases shown in (1). In order to accelerate the forecasting computation, a parameter called standardized basic level of activation is introduced (in the probabilistic price forecasting models), bl_{a_std}, for a standardized function, K_{v_std}, which corresponds to the function K_v multiplied by $h_v\sqrt{2\pi}$. Thus, the basic level of activation, bl_a, which corresponds to bl_{a_std} divided by $h_v\sqrt{2\pi}$, defines the minimum level of activation in the Gaussian kernel function K_v. Theoretically the parameter bl_{a_std} could be a value between 0 and 1, with a wide knowledge base activation for low values of bl_{a_std} and straiten knowledge base activation for values close to 1. The value of bl_{a_std}, or the corresponding bl_a, defines the values of $x_{v,new\_\min}$ and $x_{v,new\_\max}$ by using the inverse of the cumulative distribution function CK_v, of the Gaussian kernel function K_v, through (10) and (11).

$$x_{v,new\_\min} = CK_v^{-1}\left(bl_a ; x_{v,new}, h_v\right) \quad (10)$$

$$x_{v,new\_\max} = CK_v^{-1}\left(\left(1 - bl_a\right); x_{v,new}, h_v\right) \quad (11)$$

In comparison with the selection of the type of kernel function, the choice of the bandwidths, h_v (for each explanatory variable v), is more important because if the bandwidth values are relatively small, then we will obtain an underestimation of the dispersion of the estimated uncertainty of probabilistic price forecasts, with a risk of over-fitting the expected value of the hourly price (i.e., the point forecast of hourly price value) obtained from the probabilistic forecast. On other hand, if the bandwidth values are relatively large, then the dispersion of the estimated uncertainty will be excessively wide, with poor accuracy for the uncertainty of probabilistic forecasts and also for the expected value of the hourly price. The optimal bandwidth for each explanatory variable would be the one that would optimize a suitable indicator of uncertainty of probabilistic price forecasts. We select the *RI* (Reliability Indicator), presented later in Section 4.3.

The NW-KDE probabilistic forecasting tool can be applied with a dynamic iterative process to seek a value of the bandwidth h_v, for each explanatory variable v, which achieves a good value (near the optimal value) of the indicator of uncertainty of probabilistic forecast in the neighborhood of a new case $\{x_{1,new}, \cdots, x_{v,new} \cdots, x_{m,new}\}$. This dynamic iterative calibration process starts from relatively large bandwidth values, and if the indicator value improves during such a process, then all bandwidths are decreased, as described in Section 2.3.2, by using a factor F_{chg}, belonging to the interval [0,1].

An inconvenience of this dynamic iterative process would be that in some situations, the decrease of bandwidths could be excessive when activating only a very small or null number of points (cases), which would create a mathematical problem of statistical representativeness with poor representation of the probabilistic forecasting output. This inconvenience would be particularly important if the historical dataset of cases is small or if the number of explanatory variables is very large. In order to avoid this inconvenience, the NW-KDE probabilistic forecasting tool sets a parameter called "number of points", N_p, which corresponds to the minimum number of activated points from the historical dataset of cases shown in (1) , tolerable to be statistically representative for probabilistic forecasts.

2.3.2. Calibration of bandwidths

The calibration (optimization) of bandwidths h_v is carried out for each new case, basically by adjusting bandwidths for a region in the neighborhood of such a case $\{x_{1,new}, \cdots, x_{v,new} \cdots, x_{m,new}\}$, by following the stages:

Stage 1.- The iterative process starts by using a relatively large bandwidth defined for each dimension (explanatory variable v), that is 10% of (x_{v_max} - x_{v_min}), where x_{v_max} represents the maximum value of x_v of the historical dataset of cases shown in (1) and x_{v_min} represents the minimum value of such x_v .

Stage 2.- With the already defined parameters and the current values of the smoothing parameters (bandwidths), the NW-KDE probabilistic forecasting tool is applied to the historical dataset of cases (the historical dataset was indicated in (1)), which determines a set of activated points, named the "forecast set".

Stage 3.- There is a verification of the number of activated points, in accordance with the following conditions:

3.1.- If it is lower than N_p, then all the values of h_v are enlarged by the factor F_{chg}, that is, the resulting enlarged values are $h_v \cdot (1+F_{chg})$. Afterwards, return to Stage 2.
3.2.- If it is not lower than N_p, then continue to Stage 4.

Stage 4.- In the "forecast set", select the N_p points that are the "closest ones" to the new case, that is, the N_p points with the highest value of the "joint activation" computed by the expression $\prod_{v=1}^{m} K_v\left(x_{v,p}; x_{v,new}, h_v\right)$. This set of N_p points is named the "validation set" and obviously it is a subset of the "forecast set".

Stage 5.- By applying equations (6) and (7) to the "forecast set", compute the parameters α and β of the "output forecast" for the new case. From this stage, the "output parameters" (α_{new}, β_{new}, $y_{new,min}$, $y_{new,max}$) are obtained.

Stage 6.- For each case k of the "validation set", obtained in Stage 4, calculate the fitness of the historical value (y_k) in the "output forecast" which is represented by the parameters (α_{new}, β_{new}, $y_{new,min}$, $y_{new,max}$). The fitness value is computed by using the Beta cumulative distribution function F_{new} (α_{new}, β_{new}, $y_{new,min}$, $y_{new,max}$), which obtains a value in the interval [0,1]. Afterwards, the fitness values of the N_p cases in the "validation set" are used to determine the value of the *RI* (Reliability Indicator), via the process presented in Section 4.3.

6.1.- If it is the first iteration or if the value of the *RI*, for the "validation set", is better than the one obtained in the previous iteration, then all the values of h_v are reduced by the factor F_{chg}, that is, the resulting reduced values are $h_v \cdot (1-F_{chg})$. Afterwards, return to Stage 2.
6.2.- If it is not the first iteration and if the value of the *RI*, for the "validation set", does not improve with regards to the one of the previous iteration, then the process of calibration finishes.

A simplified flow chart of the proposed methodology associated to the development of the PPFM models is shown in Fig. 2.

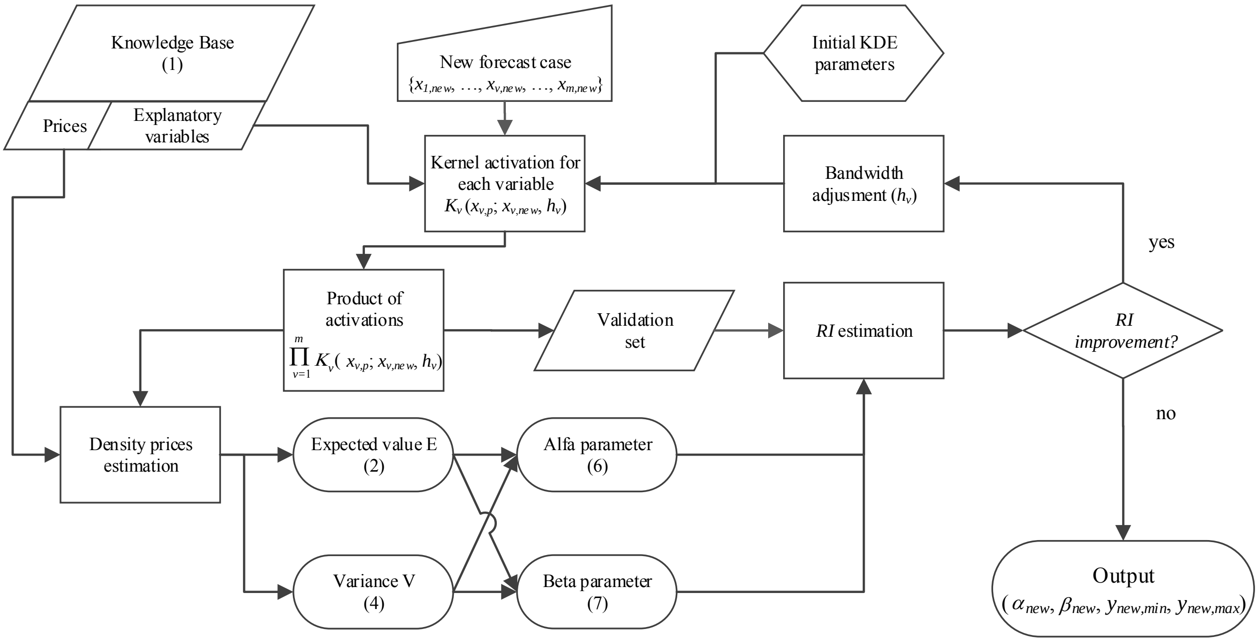

Fig. 2. Simplified flow chart of the proposed methodology associated to the development of the PPFM models.

3. Time framework and data for probabilistic price forecasting models

As abovementioned in Section 1, the real-life case study of the Iberian Electricity Market (MIBEL) was used to satisfactorily test the PPFM models, although these models can be applied to any other day-ahead electricity market. The MIBEL comprises different markets: the derivatives exchange market, managed by the company OMIP [67], and the daily and intraday markets, both managed by the company OMIE [68]. The daily market is a marginal auction market structured in a daily session, where next-day hourly sale and electricity purchase transactions are carried out.

Section 3.1 describes the time framework for the PPFM models. And afterwards, data corresponding to the real-life case study for PPFM models is shown in Section 3.2.

3.1. Time framework for probabilistic price forecasting models

PPFM models use input (price explanatory) variables that correspond to recorded hourly time series. Fig. 3 shows the time framework used for PPFM models. The day-ahead hourly price forecast $\hat{p}^{d}_{D+1,h|D,t}$ for each hour *h* of the 24 hours in day *D*+1 (output in Fig. 3) is obtained at hour *t* of the day *D*. This hour *t* of day *D* can be any moment before the closing of the daily market session and after the instant in which the variables corresponding to forecasted wind power generation and forecasted demand for the day *D*+1 are known.

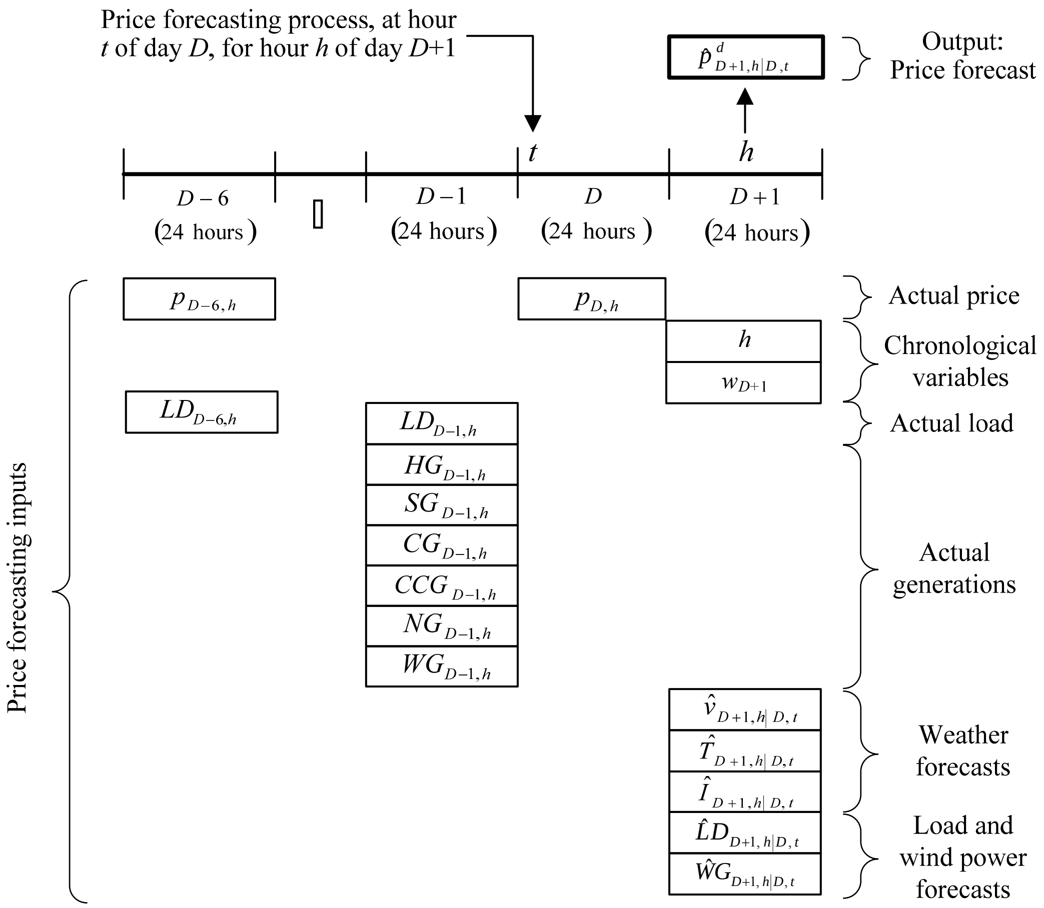

Fig. 3. Time framework of probabilistic price forecasting models.

The price $p_{D\text{-}6,h}$ for hour h of day D-6, and the price $p_{D,h}$ for hour h of the day D, are inputs to forecast the price for the hour h of day D+1. Inputs are also the day of the week w_{D+1}, the hour h of day D+1, and the weather forecasts for hour h of day D+1 (obtained at the first hours of day D for the geographical region corresponding to the electric power market), that is, regional weighted forecasted hourly temperatures $\hat{T}_{D+1,h|D,t}$, regional weighted forecasted hourly irradiations $\hat{I}_{D+1,h|D,t}$ and regional weighted forecasted hourly wind speeds $\hat{v}_{D+1,h|D,t}$.

Fig. 3 also includes other diverse inputs: hydropower generation $HG_{D\text{-}1,h}$, solar power generation and power cogeneration $SG_{D\text{-}1,h}$, coal power generation $CG_{D\text{-}1,h}$, combined cycle power generation $CCG_{D\text{-}1,h}$, wind power generation $WG_{D\text{-}1,h}$, nuclear power generation $NG_{D\text{-}1,h}$ and power demand $LD_{D\text{-}1,h}$ at hour h of day D-1; power demand $LD_{D\text{-}6,h}$ at hour h of day D-6; and wind power generation forecast $\hat{WG}_{D+1,h|D,t}$ and power demand forecast $\hat{LD}_{D+1,h|D,t}$ for hour h of day D+1.

3.2. Data for probabilistic price forecasting models

Different kinds of data have been considered for development of PPFM models. The data is as follows:

a. Chronological data (hour, day of the week).

b. Actual hourly data prices of the electricity market (MIBEL), available from the market operator OMIE [68].
c. Actual hourly data of the power system: power demand, hydropower generation, wind power generation, cogeneration and solar power generation, coal power generation, nuclear power generation, combined cycle power generation and power exchanged with France. This data was obtained by aggregating a very large amount of information from the websites of REE, the Spanish Transmission System Operator (TSO) [69], and REN, the Portuguese TSO [70].
d. Data of hourly forecasts of the power system: wind power generation forecasts and power demand forecasts. These forecasts were obtained by aggregating forecast information from the mentioned TSOs.
e. Data of hourly weather forecasts: weighted average temperature, solar irradiance and wind speed. These forecasted values were obtained with the NWP (Numerical Weather Prediction) mesoscale model WRF NMM [71], initialized with the forecasts provided by the global NWP model GFS [72]. The weighted average temperature was obtained as a weighted average of the temperature forecast for near to 250 points covering the main demand points in all the regions of the Iberian Peninsula (Spain and Portugal); the weights were proportional to the aggregated power demands of each region. Similarly, the weighted average solar irradiance and wind speed were obtained as the weighting average of hourly solar irradiances and wind speeds in nearly 750 points covering all regions of the Iberian Peninsula; in this case the weights were proportional to the regional installed capacity in solar based plants and in wind farms, respectively.

The set of price explanatory variables of the PPFM models is shown in Table 1. This set includes variables of the five kinds of data described above.

Table 1. Price explanatory variables of PPFM models.

Variable	Description
V1	hour
V2	week day
V3	hourly price *D*
V4	hourly price *D*-6
V5	hourly power demand *D*-1
V6	hourly power demand *D*-6
V7	forecasted hourly power demand *D*+1
V8	forecasted hourly temperature *D*+1
V9	forecasted hourly wind speed *D*+1
V10	forecasted hourly irradiance *D*+1
V11	forecasted hourly wind power generation *D*+1
V12	hourly wind power generation *D*-1
V13	hourly hydropower generation *D*-1
V14	hourly cogeneration and solar power generation *D*-1
V15	hourly coal power generation *D*-1
V16	hourly nuclear power generation *D*-1
V17	hourly combined cycle power generation *D*-1

The data corresponds mainly to years 2012 and 2013. It was divided into an in-sample dataset, used to create PPFM models and an out-sample dataset used for testing them. In order to have a good representation of the different price behaviors along the year, the out-sample dataset was composed of complete weeks extracted along the two years of data. The in-sample and out-sample datasets were defined as follows:

i. In-sample dataset: all the hours of the days in 2012 (from February 2nd) and 2013 (to December 4th), except those included in the out-sample dataset, totaling 13512 cases (hours). This in-sample dataset corresponds to the knowledge base dataset represented in (1).

ii. Out-sample dataset: all the hours of the weeks with numbers 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 in 2012, and weeks number 2, 7, 12, 17, 22, 27, 32, 37, 42, 47 in 2013; a total of 3360 cases (hours).

The in-sample and out-sample datasets were characterized by similar statistical parameters.

4. Application of probabilistic price forecasting models and results

Probabilistic price forecasting models (PPFM models) were applied to the abovementioned real-life case study using combinations of the explanatory variables of Table 1 and the data presented in Section 3.2.

In this Section 4, a summary of input variable selection studies for PPFM models are presented in order to determine the best combination of input variables that leads to the best PPFM model with the lowest Mean Absolute Error (MAE) value; later, detailed results of the best PPFM model for hourly price forecasts are described; lastly a Reliability Indicator (*RI*) is introduced to assess the uncertainty of probabilistic price forecasts: the *RI* provides a quantitative measure (between 0% and 100%) of "reliability" of the forecasts provided by the PPFM models.

4.1 Selection of input variables for probabilistic price forecasting models

A summary of variable selection studies for PPFM models is shown in this Section, corresponding to reasonable combinations of input variables (grouped by their common characteristics) in order to find the best PPFM model that corresponds to the input variable combination that achieves the lowest Mean Absolute Error value.

The error indicator Mean Absolute Error (MAE) was calculated, for probabilistic forecasting models, according to (12),

$$\mathrm{MAE} = \frac{1}{N}\sum_{T=1}^{N}\left|P_{forecast\_T} - P_{real\_T}\right| \qquad (12)$$

where N is the number of elements (or number of forecasting hours) in the out-sample dataset, P_{real_T} is the real hourly price value for the hour T, and $P_{forecast\_T}$ is the expected value of hourly price (point forecast of hourly price) of the PPMF model for the hour T.

The MAE errors for PPFM models (PPFM1 to PPFM23), with different input variables, are presented in Table 2. The values of the Reliability Indicator of Table 2 will be commented upon later. The selection of variables follows an ordered analysis, such that only some PPFM models are presented in the table for conclusive purposes. The construction of Table 2 corresponds to a selection process with the following sequence:

a. Models PPFM1 to PPFM3 for chronologic variables selection;
b. Models PPFM4 to PPFM6 for price variables selection;
c. Models PPFM7 to PPFM10 for power demand and forecasted power demand variables selection;
d. Models PPFM11 to PPFM16 for forecasted weather and forecasted wind generation variables selection; and
e. Models PPFM17 to PPFM23 for power generation variables selection.

Models PPFM1 and PPFM2 are two first elemental models that lead the highest MAE errors of the studied models shown in Table 2. PPFM3 is a simple baseline model with a MAE error value of 9.01 €/MWh.

Table 2. PPFM models and their MAE and Reliability Indicator values.

Type of variables	**Model**	**V1**	**V2**	**V3**	**V4**	**V5**	**V6**	**V7**	**V8**	**V9**	**V10**	**V11**	**V12**	**V13**	**V14**	**V15**	**V16**	**V17**	**MAE (€/MWh)**	**Reliability indicator (%)**
Chronologic	**PPFM1**	X																	9.78	78.51
	PPFM2		X																10.40	75.25
	PPFM3	X	X																9.01	82.51
Price	**PPFM4**	X	X	X															7.73	80.61
	PPFM5	X	X		X														9.09	79.33
	PPFM6	X	X	X	X														7.70	82.38
Demand	**PPFM7**	X	X	X	X	X													7.58	80.92
	PPFM8	X	X	X	X		X												7.60	81.86
	PPFM9	X	X	X	X			X											7.57	81.21
	PPFM10	X	X	X	X	X		X											7.48	82.98
Weather	**PPFM11**	X	X	X	X	X		X	X										7.48	83.61
	PPFM12	X	X	X	X	X		X		X									6.16	83.39
	PPFM13	X	X	X	X	X		X			X								7.52	83.17
	PPFM14	X	X	X	X	X		X	X	X									6.04	84.92
Power Generation	**PPFM15**	X	X	X	X	X		X	X	X		X							5.81	83.53
	PPFM16	X	X	X	X	X		X	X			X							5.65	85.07
	PPFM17	X	X	X	X	X		X	X			X	X						5.59	84.47
	PPFM18	X	X	X	X	X		X	X			X		X					5.58	85.25
	PPFM19	X	X	X	X	X		X	X			X			X				5.63	84.84
	PPFM20	X	X	X	X	X		X	X			X				X			5.62	85.75
	PPFM21	X	X	X	X	X		X	X			X					X		5.60	84.11
	PPFM22	X	X	X	X	X		X	X			X						X	5.62	83.65
	PPFM23	X	X	X	X	X		X	X			X		X		X	X	X	5.55	85.64

If variable V3 (hourly price *D*) is added to those used by model PPFM3, leading to model PPFM4, then the MAE error decreases to 7.73 €/MWh; and model PPFM6 obtains an error of 7.7 €/MWh when including the price *D*-6 variable (V4) in model PPFM4.

The inclusion of variable V5 (hourly power demand *D*-1) in model PPFM6 slightly reduces the MAE error to 7.58 €/MWh (model PPFM7). Additionally, if variable V7 (hourly forecasted power demand *D*+1) is included in model PPFM7, the MAE error is reduced to 7.48 €/MWh (model PPFM10).

From the point of view of the set of models PPFM11 to PPFM14, compared to model PPFM10, the forecast wind speed variable (V9) in model PPFM12 obtains a MAE error of 6.16 €/MWh. Furthermore adding the forecasted temperature variable (V8) in model PPFM14 leads to a slightly better error of 6.04 €/MWh.

The forecasted wind power variable (V11) added to model PPFM14 achieves an error of 5.81 €/MWh (model PPFM15). Since variables V9 and V11 have collinear information, the exclusion of variable V9 in model PPFM15 leads to a slightly better error of 5.65 €/MWh (model PPFM16).

Models PPFM17 to PPFM22 allow the evaluation of the improvement in the MAE error by adding variables V12 to V17 (power generation variables *D*-1). Variable V13 (hydropower generation *D*-1) is the one that achieves a lower error, 5.58 €/MWh in model PPFM18. The inclusion of variables V15 to V17 (thermal power generation *D*-1) in the previous model PPFM18 results in the best performance of PPFM models (model PPFM23), reaching a MAE error value of 5.55 €/MWh. Therefore, among models PPFM1 to PPFM23, the best PPFM model is PPFM23.

Notice that there are five families of PPFM models (which correspond to the five families of types of price explanatory variables): i) models PPFM1, PPFM2 and PPFM3, containing chronological variables; ii) models PPFM4, PPFM5 and PPFM6, which include price variables (and chronological ones); iii) models PPFM7 to PPFM10, which contain demand variables (and price and chronological ones); iv) models PPFM11 to PPFM14, which take into account weather forecasting variables (and demand, price and chronological variables); and v) PPFM15 to PPFM23, which include generation variables (and weather forecasting, demand, price and chronological variables).

Lastly, observe that, from the point of view of the MAE error values in Table 2, the best model among models PPFM1, PPFM2 and PPFM3 is model PPFM3. MAE errors of models PPFM4, PPFM5 and PPFM6 in Table 2, determine that the best model is PPFM6. The best model, upon analyzing MAE errors of models PPFM7 to PPFM10, is model PPFM10. Among the models PPFM11 to PPFM14, the best model is PPFM14. Lastly the best model upon studying MAE errors of models PPFM15 to PPFM23, is model PPFM23. Thus, the best models of each of the five families of probabilistic price forecasting models are models PPFM3, PPFM6, PPFM10, PPFM14 and PPFM23.

4.2 Results of the best probabilistic price forecasting model PPFM23

Values of the input variables used in model PPFM23 (the best probabilistic price forecasting model among models PPFM1 to PPFM23) for three forecasts of hourly price are shown in Table 3, as well as output (results) values, quantiles and the real price values. First forecast (Forc1) corresponds to the nineteenth hour on a Sunday (18:00 to 18:59 on May 26th, 2013), on a day without significant renewable generation but with relatively low prices on the previous day, resulting in a normal price at a time like this on a Sunday. Second forecast (Forc2) refers to the twenty-third hour on a Thursday (22:00 to 22:59 on May 30th, 2013), on a working day but with significant wind and hydro generation, resulting in a relatively low price at a time like this on a Thursday. Third forecast (Forc3) was obtained for the eleventh hour on a Monday (10:00 to 10:59 on October 14th, 2013), on a working day with low wind and hydro generation and with relatively high prices on previous days, resulting in a high price at a time like this on a Monday.

Thus, the expected values of hourly price (i.e. point forecasts of price) obtained from the model PPFM23 correspond to price values that are relatively close to real prices values, since the percentages of error were 1.6%, -6.6% and -0.7% for the forecasts Forc1, Forc2 and Forc3, respectively (Table 3), in which such error percentages were calculated using the difference between expected and real price values with respect to the real ones.

In Table 3, the variance values correspond to a comparatively higher dispersion of the probability density functions of forecasts Forc1 and Forc2, and to a relatively svelter probability density function of the forecast Forc3.

Table 3. Forecasts from the probabilistic price forecasting model PPFM23

		Forc1	**Forc2**	**Forc3**
Input variables	V1 hour (0-23h)	18	22	10
	V2 week day (1-7)	1	5	2
	V3 price *D* (€/MWh)	30.86	46.20	74.20
	V4 price *D*-6 (€/MWh)	52.76	51.10	68.96
	V5 power demand *D*-1 (MWh)	29457	36760	26392
	V7 forecasted power demand *D*+1 (MWh)	35055	34732	36963
	V8 forecasted temperature *D*+1 (ºC)	16.3	18.1	15.4
	V11 forecasted wind power *D*+1 (MWh)	7433	11979	8467
	V13 hydropower generation *D*-1 (MWh)	2093	8081	1200
	V15 coal power generation *D*-1 (MWh)	4277	4755	8017
	V16 nuclear power generation *D*-1 (MWh)	4691	4077	5994
	V17 combined cycle power *D*-1 (MWh)	1768	2029	2612
Real value	Real price (€/MWh)	47.23	46.75	59.05
Outputs	Expected value of price (€/MWh)	47.97	43.68	58.66
	Variance $(€/MWh)^2$	19.22	31.33	9.81
	Parameter α	5.739	3.532	3.165
	Parameter β	6.534	6.694	2.139
	Minimum price (€/MWh)	33.00	30.06	49.10
	Maximum price (€/MWh)	65.01	69.50	65.13
	Percentage error (%)	1.6	-6.6	-0.7
Quantiles	Quantile Q10 (€/MWh)	42.25	36.64	54.31
	Quantile Q25 (€/MWh)	44.83	39.50	56.42
	Quantile Q50 (€/MWh)	47.91	43.27	58.87
	Quantile Q75 (€/MWh)	51.05	47.45	61.10
	Quantile Q90 (€/MWh)	53.77	51.31	62.70

In Table 3, values of results (parameters α and β, and maximum and minimum price) determine probability density functions (corresponding to Beta distributions), which allow a variety of useful information to be obtained, such as quantile representations and probability calculations. For instance, we can directly answer the question: "What are the probabilities that electricity prices will be higher than 52 €/MWh?"; they are 19%, 8% and 98% for the forecasts Forc1, Forc2 and Forc3, respectively. These probability density functions can also be used to obtain quantiles. As such, Table 3 displays quantiles Q10, Q25, Q50, Q75 and Q90 for the three forecasts of hourly price. Observe that quantile Q50 is relatively close to the expected hourly price value. Furthermore, such quantiles are also going to be shown later, in time series representations of the forecast uncertainty, throughout two weeks (Fig. 4 and Fig. 5).

Fig. 4 shows the representation of the probability density functions for the three probabilistic price forecasts Forc1, Forc2 and Forc3. The probability density function (PDF) for forecast Forc1 is more symmetrical (the value of the parameter β is relatively close to that of the parameter α). On one hand, the PDF function for forecast Forc2 is slightly right tailed because of the higher value of the parameter β with respect to that of the parameter α. On other hand, the PDF function for forecast Forc3 is slightly left tailed because of the higher value of parameter α in comparison with that of the parameter β. It is also possible to verify that the real values of the price are "inside" the probability density functions for these forecasts: more specifically, the real price values are situated within the percentiles 44%, 71% and 52% for forecasts Forc1, Forc2 and Forc3, respectively.

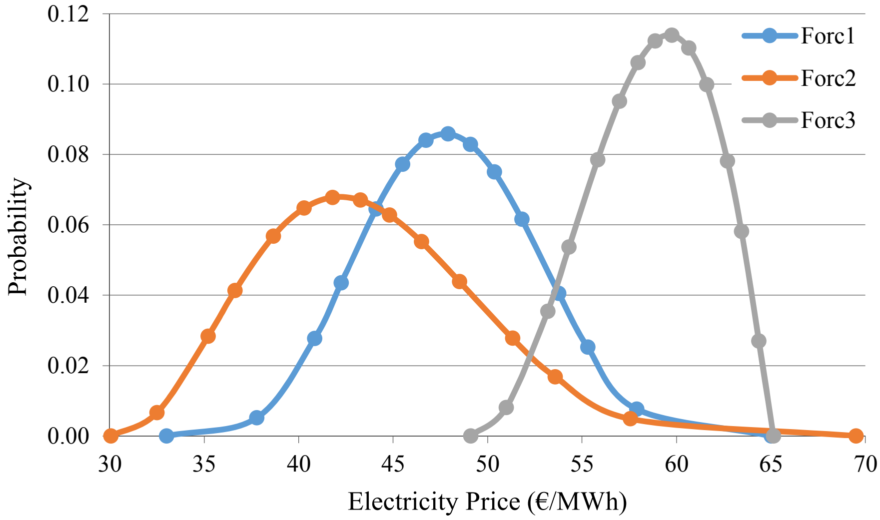

Fig. 4. Probability density functions for forecasts of hourly price.

Lastly, Fig. 5 and Fig. 6 give results (time series representations of the forecast uncertainty) from the probabilistic price forecasting model PPFM23 for week number 22 in May 2013, and for week number 42 in October 2013, showing quantiles and also real prices.

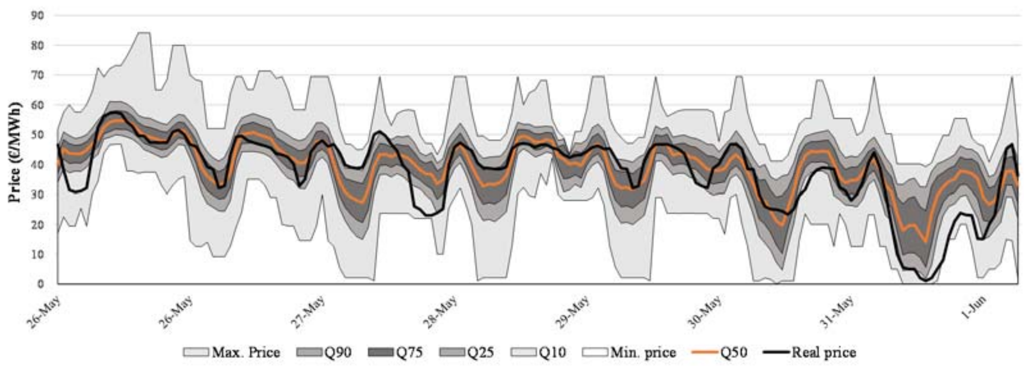

Fig. 5. Probabilistic price forecasts for week number 22 in May 2013

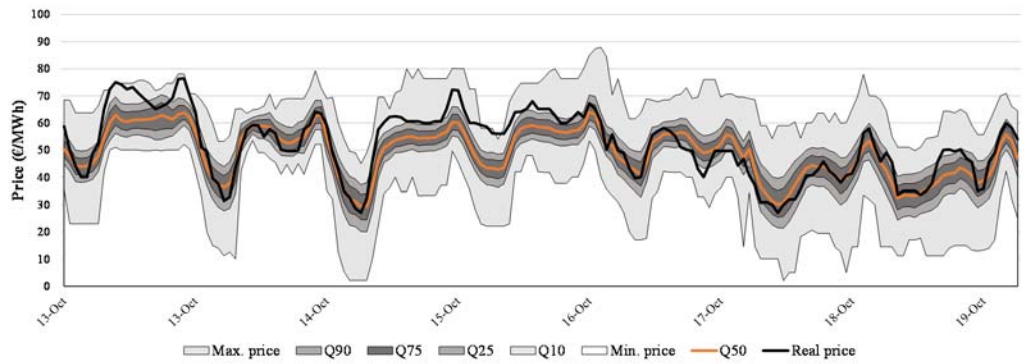

Fig. 6. Probabilistic price forecasts for week number 42 in October 2013

4.3 Indicator of probabilistic price forecast uncertainty of PPFM models

The accuracy of explainable information of the forecasts of the PPFM models can be evaluated based on deviations of the real price values with respect to the expected values of the hourly price (point forecast of hourly price) from such models, by using appropriate performance error indicators as the indicator MAE above mentioned.

Indicators of uncertainty appropriate for probabilistic price forecasts have to be able to evaluate the accuracy of the forecasts of probabilistic forecasting models from the point of view of the forecast uncertainty. In this aspect, a suitable reliability diagram can be used to analyze the "reliability" associated to forecasts of PPFM models.

Fig. 7 shows a reliability diagram associated to an example of forecasts of the probabilistic model PPFM23. In the vertical axis of the figure, the diagram gives the frequency of the events (forecasted price occurrences) for the cases of the out-sample dataset, in each partition of the "normalized" scale (between 0 and 1) of fitness values of price, that is shown in the horizontal axis. The expected value of the hourly price, for each new case q of the out-sample dataset, is obtained by using the parameters (α_{new_q}, β_{new_q}, $y_{new\_q,min}$, $y_{new\_q,max}$) of a probability density function (PDF) of a Beta distribution. Afterwards, the fitness value of the corresponding historical value of the hourly price (y_q) is calculated. This fitness value is computed by using the Beta cumulative distribution function

$F_{new\_q}\,(y_q;\ \alpha_{new\_q},\ \beta_{new\_q},\ y_{new\_q,min},\ y_{new\_q,max})$, which obtains a value in interval [0,1], to be situated in the horizontal axis of Fig. 7. These fitness values are classified in the twenty partitions, with intervals of 0.05. The twenty partitions presented in intervals of 0.05, on the horizontal axis of the figure, have an ideal frequency ("target frequency") of 5%. Since these partitions correspond to a bounded interval [0, 1], the target frequency for the two additional partitions outside such interval must be 0%. Fig. 7 shows that there is about 2% of price occurrences (events) in the partition corresponding to indication "< MIN", i.e. below the interval [0, 1], and there is about 3% of price occurrences in the partition corresponding to indication "> MAX", i.e. above such interval. Most of the partitions between 0.05 and 0.35 on the horizontal axis correspond to frequencies of price occurrences that are clearly below the target frequency. In the partition 0.95-1.00, obviously the frequency of price occurrences is above the target frequency.

A suitable indicator to evaluate the "reliability" value associated to forecasts of PPFM models, the so-called Reliability Indicator, *RI*, was introduced. It gives a measure related to the average normalized difference between the observed frequencies and target frequency, according to (13),

$$RI = \left(1 - \sum_{i=1}^{NQ} \left| f_{obs,i} - f_{tar,i} \right| \right) \cdot 100\% \qquad (13)$$

where $f_{obs,i}$ and $f_{tar,i}$ are the observed frequencies (in per unit) and the target frequency (in per unit) for the interval i ; and NQ is the total number of partitions (intervals).

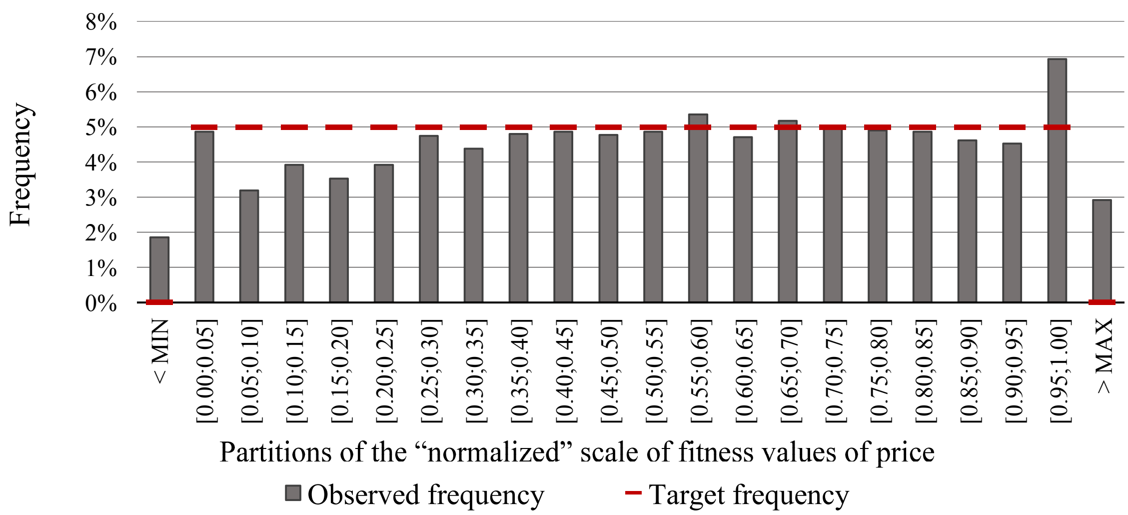

Fig. 7. Reliability diagram example from the probabilistic price forecasting model PPFM23.

The reliability diagram of Fig. 7, obtained from an example of forecasts of the model PPFM23, corresponds to a value of the indicator *RI* of 84.2%. The values of the *RI* for probabilistic models PPFM1 to PPFM23, applied to price forecasts using the data of Section 3.2, are given in Table 2.

Observe that the best model PPFM23, which achieved the lowest MAE error, obtained one of the highest *RI* values.

5. Discussion

As indicated in Section 4, probabilistic price forecasting models PPFM3, PPFM6, PPFM10, PPFM14 and PPFM23, are the best models of each of the five families of PPFM models.

Fig. 8 shows values of two indicators (the MAE and the *RI*) for forecasts of the probabilistic price forecasting models PPFM3, PPFM6, PPFM10, PPFM14 and PPFM23, using the data of Section 3.2. In Fig. 8, the MAE error improves (decreases) as more types of input variables (chronological variables, price variables, power demand variables, forecasted weather variables, and power generation variables) are included in PPFM models: the integration of price variables in model PPFM6 (variables not included in model PPFM3) and the integration of meteorological variables in model PPFM14 (variables not included in model PPFM10) represent the information that causes a greater improvement of the MAE indicator. The integration of demand variables in model PPFM10 (variables not included in model PPFM6) constitutes the information that causes a comparatively lower improvement of the MAE indicator.

Fig. 8 shows that the *RI* (indicator used to assess the uncertainty associated to probabilistic price forecasts) improves as more types of explanatory input information (chronological information, prices, power demands, forecasted weather information, and power generations) are included in PPFM models, with the exception of the inclusion of price information in model PPFM6 (information not included in model PPFM3), since in this case the *RI* does not increase. The most important improvement in the *RI* is obtained when including the forecasted weather information in model PPFM14 (information not included in model PPFM10).

Therefore, the MAE error values improved and the *RI* ("reliability") values habitually also improved as more types of input variables (chronological variables, price variables, power demand variables, forecasted weather variables and power generation variables) were included in such PPFM models.

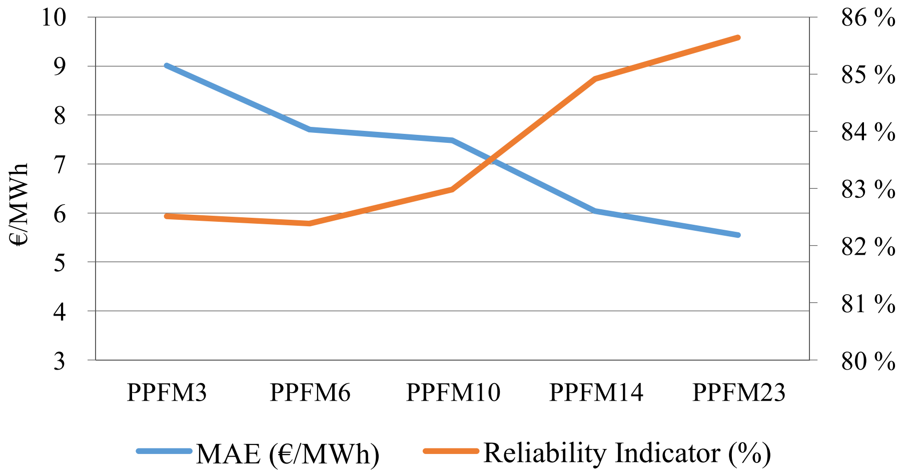

Fig. 8. Comparison of performances of probabilistic price forecasting models.

6. Conclusions

Hourly price forecasts for electric energy, in an electricity market, constitutes very useful information if known in advance, because any agent involved in such market can use such information to made strategic bids in order to maximize the associated economic profit. Furthermore, obtaining accurate price forecasts has direct impact on the producers' electric energy management. This is why electricity price forecasting has been a very active research field in the last 15 years.

Day-Ahead Electricity Price Forecasting models have been presented in the scientific literature in recent years, mostly centered on spot (point) forecasts of hourly prices, but unfortunately they can be inadequate for trading purposes because they do not show the uncertainty associated with predicted price values, that is, spot forecasts do not provide any fundamental information for risk-based market decisions. Therefore, from this point of view, probabilistic models for price forecasting overcome the limitations of spot forecasts. The most important probabilistic approaches are focused on probability density functions (PDFs). However, there is a scarcity of probabilistic approaches based on PDFs.

This article presents original Probabilistic Price Forecasting Models (PPFM models) for the day-ahead hourly probabilistic price forecasting using probability density functions of Beta distributions. The PPFM models are based on the Nadaraya–Watson Kernel Density Estimator (NW-KDE) using a suitable Gaussian KDE function for each one of its input variables, i.e., for each one of the explanatory variables of the electricity price. The new PPFM models use a very extensive set of input variables, consisting of large time series of hourly prices in previous days, regional-aggregated hourly power generations in the previous day (hydropower generation, wind power generation, solar power generation and power cogeneration, nuclear power generation, combined cycle power generation, and coal power generation), hourly power demand in the previous day, and in the previous week. Furthermore, input variables include forecasts of regional-aggregated hourly power demands, forecasts of wind power generation and weather forecasts (hourly wind speed, temperature, and irradiation) in the region for the day-ahead, as well as chronological data.

The PDF of a Beta distribution for each hour of the price variable (output variable), for each PPFM model, is directly obtained from the expected and variance values associated with the NW-KDE approach. Hence, the direct use of the historical input dataset (historical cases) without simplifications or information losses in the knowledge base (very extensive input datasets of the PPFM models) is particularly important when the purpose is to extract fundamental probabilistic information from historical similar cases.

As indicated, the uncertainty representation of the output (price variable) in each forecast hour of each PPFM model is given by a parametric Beta distribution defined by four parameters directly obtained from the NW-KDE approach. From these Beta distribution parameters we calculate, in a straightforward way, diverse types of probability values, e.g. the probabilities that electricity prices will be higher or lower than a certain threshold, prediction intervals, and any kind of quantiles. The aforementioned Beta PDF also provide powerful probabilistic information for risk-based decisions instead of point forecasts useful only for simple decisions. In electricity market business, when the market agent takes his or her decisions according to risk-based criteria, probabilistic information is essential for agent decisions involving different markets or products (e.g. daily spot, intraday spot, bilateral, or futures markets) with different levels of uncertainty. Parametric probabilistic price forecasts, efficiently used, provide a very significant advantage for market agents implicated in risky decision-making business processes, especially when high uncertainty of the prices occurs. Compared to other representations of uncertainty (e.g. quantile representations), the parametric representation approach used in the PPFM models of this article provides high adaptability to the risk-based analysis (decision processes, based on forecasts of the PPFM models, are not conditioned by the discretization of quantile representations). Furthermore, the parametric approach of Beta distributions obtained from the PPFM models surpasses the simple interval representations of the uncertainty.

A new Reliability Indicator (*RI*) allows the uncertainty associated with price forecasts of each PPFM model to be evaluated; i.e., the *RI* gives a quantitative measure (from 0% to 100%) of “reliability” of the forecasts of each PPFM model. A suitable representation of a “reliability diagram” (associated with the *RI*) is presented in this article, designed to better evaluate the performance of any PDF of Beta distribution. Such suitable “reliability diagram” identifies “forecast cases” that fall outside the limits of the PDF. Then, the performance of a Beta distribution can be studied directly by exploring the diagram. On one hand, the *RI* provides performance comparisons of the PPFM models from the point of view of “reliability”. On the other hand, the *RI* is used in a novel procedure of bandwidths calibration, outlined below.

An original procedure is used to calibrate the bandwidths of the kernel Gaussian functions involved in the NW-KDE approach. The dynamic calibration process of a bandwidth is carried out by adjusting its value for each “forecast case” (“new case”), mainly based on the inputs and on the similarity of historical cases in the neighborhood of this new case to be forecast. This dynamic adjustment betters (as much as possible) the bandwidths by controlling the improvement of a reliability measure given by the *RI*.

The new PPFM models were, for the first time, successfully applied to the real-life case of the Iberian Electricity Market (MIBEL) that covers mainland Portugal and Spain, although these PPFM models can be applied to any other day-ahead electricity market. A systematic analysis of the MAE errors of PPFM models corresponding to suitable combinations of their input variables was carried out in order to determine the best PPFM model, that is, the best combination of input variables for PPFM models that achieves the lowest MAE error value. Multiple reasonable combinations of input variables were analyzed, from simple naïve PPFM models (only with chronological input data) to much more complex PPFM models, with up to 17 input variables of data which include historical values of prices, power generations, demands, and forecasted values of demands, wind speeds, temperatures and irradiations, as well as chronological data. The best PPFM model was the model PPFM23 which included 12 of these input variables and achieved a MAE of 5.55 €/MWh.

The accuracy of explainable information of the forecasts of the PPFM models was evaluated by the indicator MAE; and the uncertainty of the price forecasts obtained by PPFM models was evaluated by the *RI*. Comparisons of performances of PPFM models led to the conclusion that the MAE error values improved and that the *RI* ("reliability") values usually also improved as more types of input variables (chronological variables, price variables, power demand variables, forecasted weather variables, and power generation variables) were included in such PPFM models.

The probabilistic price forecasting models of this article, their performance, mainly in terms of MAE error and Reliability Indicator values, pragmatic calculations applied to the PDF functions associated to day-ahead probabilistic price forecasts, and the best PPFM model, can be useful for business players within electrical energy markets and other agents from the electric power industry.

Possible future research works related to this article would probably be aimed to improve the bandwidth calibration process, to use other probabilistic forecasting scores, and to analyze the performance of the PPFM models using a non-Gaussian KDE.

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