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# Computer simulation of the laminar nozzle flow of a non-Newtonian fluid in a rubber extrusion process by the finite volume method and experimental comparison

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**Abstract**

The aim of this work is to describe the computer simulation of the laminar flow through a nozzle in a rubber extrusion process by the finite volume method (FVM). The liquid rubber is a highly compliant material, whose behavior is not described by the Newtonian constitutive relations, and whose underlying physics is not yet completely understood. The processing and transport of such fluids are central problems in the polymer, plastics and automotive industries. Non-Newtonian behavior manifests itself in a number of different ways. This fluid exhibits a shear rate dependent viscosity, with 'shear thinning', that is, decreasing viscosity with increasing shear rate, being the most prevalent behavior. We have taken the power-law model in order to simulate this rubber extrusion process, which has the form $\mu = KI_2^{(n-1)/2}$, where $\mu$, $I_2$, $n$ and $K$ are termed the dynamic viscosity, the second invariant of the rate of deformation tensor, the power-law index and the consistency, respectively. These last two parameters were obtained from experimental tests and used in a computer simulation. In this work we have modeled two types of rubbers and different inlet pressures, for a type of nozzle, in order to calculate the outlet velocity distribution of the rubber jet in this extrusion process. Finally we have compared the numerical and experimental results, so that this model is consistent with the experimental evidence.


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## 1. Introduction

Materials encountered in industry invariably fall outside the classical extremes of the Newtonian viscous fluid and Hookean elastic solid. When such materials can be classified as fluids, the adjective 'non-Newtonian' is usually employed [1]. This paper presents the numerical study by the finite volume method (FVM) [2] of non-Newtonian fluid in a rubber extrusion process and its experimental validation.

To be more precise, we define a non-Newtonian fluid [3] to be one whose behavior cannot be predicted on the basis of the Navier–Stokes equations. Such fluids may or may not posses a memory of past deformation. In the case of rubber the memory effects are not significant, that is to say, is an 'inelastic fluid' [4,5]. Basically, inelastic fluids can be viewed as generalizations of the Newtonian fluid.

The viscosity function for such materials depends on the rate of deformation of the fluid and thus allows 'shear thinning' effects to be modeled [6].

In development and use, non-Newtonian fluids often encounter complex geometries [1]: lubricants have to operate in gears and bearings; molten polymers meet complex geometries with and without free-surface complications in

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injection molding and similar processes and, finally, in this work a liquid rubber has to be 'squeezed' through a contraction at the exit of a nozzle.

## 2. Power-law fluid

### 2.1. Constitutive equation

The form of the constitutive equation for a generalized Newtonian fluid was given as [5,6]:

$$\sigma_{ij} = -P\delta_{ij} + 2\mu(D_{ij})D_{ij}, \quad (1)$$

where $\sigma_{ij}$ are the components of the total stress tensor, $P$ is the pressure, $\delta_{ij}$ is the Kronecker delta, and $D_{ij}$ are the components of the rate of deformation tensor. Since the viscosity $\mu$ for non-Newtonian fluids depends on $D_{ij}$, that is, of its invariants, defined by [7,8]

$$I_1 = \mathrm{tr}(D); \quad I_2 = \frac{1}{2}\mathrm{tr}(D^2); \quad I_3 = \frac{1}{3}\mathrm{tr}(D^3), \quad (2)$$

where tr denotes the trace. There is theoretical and experimental evidence to suggest that the viscosity depends only on $I_2$ as $\mu = \mu(D_{ij}) = \mu(I_2)$.

The rubber in liquid state shows a non-Newtonian behavior. In this work, we have modeled this behavior according to power-law model, which has the form [1,3–6]

$$\mu = KI_2^{(n-1)/2}, \quad (3)$$

where $n$ and $K$ are parameters termed the *power-law index* and *consistency*, respectively. The liquid rubber in the extrusion process has an index $n < 1$. This fluid is termed *shear thinning* or *pseudoplastic* [3]. The admissible range of shear rate for this liquid rubber ranges from 10 to 100 $s^{-1}$.

### 2.2. Experimental parameters

From different test carried out by the BTR Sealing Systems (Germany) on the real liquid rubbers, we have obtained the power-law parameters fitting the experimental data to a curve of potential kind (see Fig. 1).

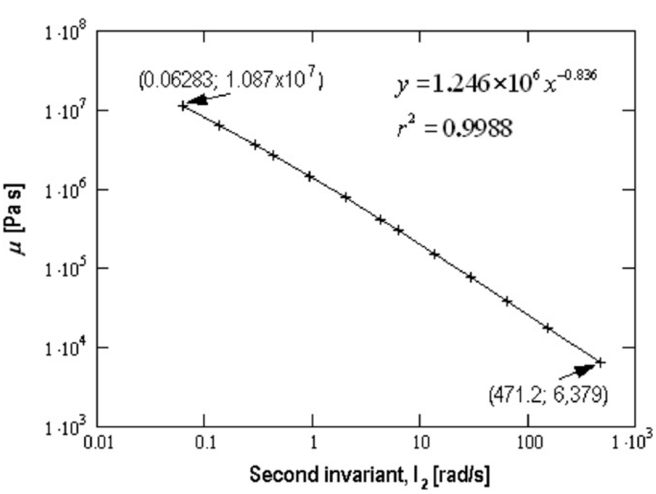


Fig. 1. Experimental data, potential fitting ($y$) and correlation coefficient ($r^2$).

## 3. Mathematical model

The finite volume formulation [2] of non-Newtonian flows follows very closely the formulation developed for Newtonian flow problems [9,10]. Therefore, in the present work only a brief overview of the general formulation will be given, with more attention focused on those aspects that are unique to the non-Newtonian problem.

To simulate the extrusion process, we have used the finite volume method (FVM) [2]. The finite volume method (FVM) is one of the most versatile discretization techniques used in computational fluid dynamics (CFD). Based on the control volume formulation of analytical fluid dynamics, the first step in the FVM is to divide the domain into a number of control volumes (see Fig. 2) where the variable of interest is located at the centroid of the control volume. The next step is to integrate the differential form of the governing equations over each control volume [11]. Interpolation profiles are then assumed in order to describe the variation of the concerned variable between cell centroids. The resulting equation is called the discretized equation. In this manner, this equation expresses the conservation principle for the variable inside the control volume [12].

The most important feature of the FVM is that the resulting solution satisfies the conservation of quantities such as mass, momentum and energy. This is exactly satisfied for any control volume as well as for the whole computational domain and for any number of control volumes. Even a coarse grid solution exhibits exact integral balances.

The approach used in the ANSYS-CFX code is the FVM. The governing equations are solved on discrete con-

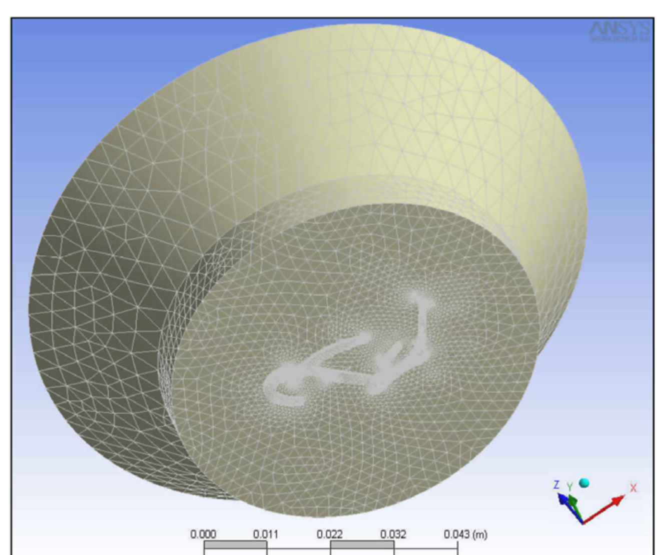


Fig. 2. Detail of the volume finite mesh.

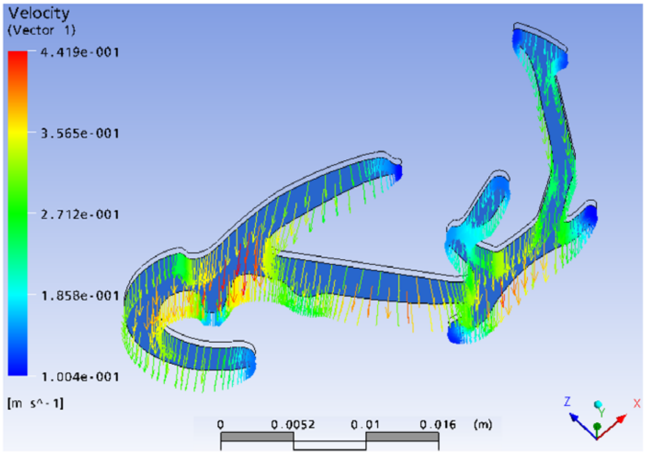


Fig. 3. Velocity vector diagram in the nozzle's outlet.

trol volumes. This integral approach yields a method that is inherently conservative [2,12]

$$\frac{\partial}{\partial t}\iiint Q\,\mathrm{d}V + \iint F\,\mathrm{d}A = 0, \tag{4}$$

where $Q$ is the vector of conserved variables, $F$ is the vector of fluxes, $V$ is the cell volume, and $A$ the cell surface area [2]. This set of equations is virtually identical to those used in Newtonian problems except for the dependence of the viscous diffusion term on the velocity, i.e., the rate of deformation [5] (because of viscosity's dependence on the rate of deformation tensor), and possibly the temperature.

## 4. Results and conclusions

The finite volume method along with data from real tests carried out in a rubber extrusion process is used for comparison purposes. The findings of this study suggest that it may be possible to devise a practical procedure for establishing a quality model by using a combined experimental/computational approach.

From the experimental measurements it is observed that the velocity in the nozzle's outlet ranges from 0.17 to 0.33 m/s and from 0.1 to 0.44 m/s for the numerical results. This speed variation across this section requires geometry modifications by opening angles on rubber channels where the speed is low, in order to improve its speed.

The variation of the velocity field is similar in both methods. There is a good agreement between FVM numerical results and experimental test (see Fig. 3).

In this work we have used a tetrahedral unstructured mesh with refining around the nozzle's outlet surfaces, the element sizes used vary from 0.0005 m in the near field to 0.001 m for the far field. Our solver can be run so as to preserve time accuracy or as a pseudo-unsteady formulation to enhance convergence to steady state. It uses dynamic memory allocation, since the problem size is only limited by the amount of memory available on the machine.

Finite volume procedures are at present very widely used in engineering analysis, and we can expect this use to increase significantly in the years to come. The FVM [2,11–13] is a robust and cheap method for the discretization of conservation laws. By robust, we mean a numerical scheme which behaves well even for particularly difficult equations: non-linear systems of hyperbolic equations, like in this case.

We have fitted the experimental data obtained from BTR Sealing Systems Company to a curve of potential kind. The liquid rubber exhibited a strongly non-Newtonian behavior, showing a substantial decrease in viscosity with increasing shear. Therefore, in this work we have modeled the rubber extrusion process by means of a power-law model, which can be used to improve the solid rubber quality obtained in this industrial process.

The above analysis contribute to a comprehensive understanding of the non-Newtonian flow behavior and of the so called 'gross' shear thinning effect of amorphous polymers such as liquid rubbers.

Finally, in future works we shall also apply this technique to the non-isothermal fluid problem.

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## References

[1] M.J. Crochet, A.R. Davies, K. Walters, Numerical Simulation of Non-Newtonian Flow, Elsevier Science, Amsterdam, 1984.
[2] R.J. Leveque, Finite Volume Methods for Hyperbolic Problems, Cambridge University, New York, 2002.
[3] D.A. Siginer, D. De Kee, R.P. Chhabra, Advances in the Flow and Rheology of Non-Newtonian Fluids, Elsevier Science, Amsterdam, 1999.
[4] J.N. Reddy, An Introduction to Non-Linear Finite Element Analysis, Oxford University, Oxford, 2005.
[5] J.N. Reddy, D.K. Gartling, The Finite Element Method in Heat Transfer and Fluid Dynamics, CRC, New York, 1994.
[6] C. Dafermus, J.L. Ericksen, D. Kinderlehrer, Amorphous Polymers and Non-Newtonian Fluids, Springer, New York, 1987.
[7] L.M. Milne-Thomson, Theoretical Hydrodynamics, Dover, New York, 1996.
[8] P. Chadwick, Continuum Mechanics: Concise Theory and Problems, Dover, New York, 1999.
[9] G.K. Batchelor, An Introduction to Fluid Dynamics, Cambridge University, New York, 2000.
[10] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, New York, 1987.
[11] B.R. Munson, D.F. Young, T.H. Okiishi, Fundamentals of Fluid Mechanics, John Wiley, New York, 2005.
[12] I. Shames, Mechanics of Fluids, McGraw-Hill Science, New York, 2002.
[13] R.W. Fox, Introduction to Fluid Mechanics, Wiley, New York, 2003.