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A Hybrid Equilibrium in Segmented Markets: the Three-Firm Case

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In this paper, we characterize two hybrid equilibria for the three-firm case in segmented markets in which consumers not only value the product itself but also the environment within which the consumption takes place. In equilibrium, the firm with the larger population of loyal consumers chooses the monopoly price while the remaining two firms play a mixed strategy. In the duopoly case, the unique equilibrium is in mixed strategies and no firm focuses only on its loyal consumers.

Keywords: segmentation, loyalty, mixed strategy.

JEL classification: D43, L13.

1 Introduction

In this paper, we are concerned with price competition in segmented markets.[1] We focus on markets in which consumers not only value the product they are consuming but also the seller they are buying from. There are circumstances under which this behavior is realistic. In markets where domestic firms compete with foreign ones, some consumers may exhibit nationalistic preferences and buy from the domestic producer even though the foreign one sells at a lower price (American cars versus Japanese cars, domestic airline versus foreign one, etc.). In markets with one incumbent

1 Shilony (1977) was the first to analytically examine price competition in segmented markets. Varian (1980) used a variation of Shilony's model to analyze the rationale of sales practices in retail markets. Wilde and Schwartz (1979) analyzed price dispersion in markets in which consumers had different attitudes towards searching for information about prices. For models of price dispersion, see also Salop and Stiglitz (1977), and Stiglitz (1979).

firm, consumers may become loyal to the firm just by trying its product first. Potential entrants will have to focus on those consumers who compare prices. There are also products whose consumption takes place in environments that affect the utility enjoyed by other consumers. Some consumers may refrain from consuming if the environment does not fit with their characteristics, or only consume if the environment is to their taste.

We consider a model of horizontal product differentiation that captures these features.[2] In particular, we analyze price competition by three rival cafés with different smoking policies when the consumers do not only differ in smoking habits but also in their attitudes towards comparing prices. Consumers who smoke do not patronize smoke-free cafés. Nonsmokers are divided between those who do not visit cafés that allow smoking (radical nonsmokers) and those who, regardless of smoking policies, compare prices and choose the café with the lower price (nonradical nonsmokers). The existence of price-sensitive consumers presents a dilemma to the cafés: they have to decide whether to compete for them or focus only on their loyal consumers.

We find that there are qualitative differences in the nature of the equilibria when we consider more than two cafés. While under duopoly, cafés compete for the price-sensitive consumers and the unique equilibrium is in mixed strategies, hybrid equilibria arise with three or more cafés where one café plays a pure strategy (the monopoly price) and the remaining cafés play a mixed strategy. With more than two cafés, competition for the consumers who compare prices is more intense. Some cafés prefer to focus on their loyal consumers and exploit them by charging the monopoly price.

In Sect. 2, we describe the model and provide a characterization of the unique equilibrium in the duopoly case. In Sect. 3, we show that hybrid equilibria arise when we consider three or more cafés. Finally, some conclusions are presented.

2 The Model with Two Cafés

We consider a model of duopolistic competition between two rival cafés with different smoking policies. Café 1 allows smoking while café 2 is smoke-free. We normalize the size of the population of consumers to one. All consumers have a common reservation price r for a cup of coffee.

2 For spatial-differentiation models, see Hotelling (1929), d'Aspremont et al. (1979), Salop (1979), and Economides (1986).

Let $s \in (0, 1)$ be the population of smokers. Smokers do not patronize café 2.[3] They buy at café 1 if its price is not higher than their reservation price. Let $1 - s \in (0, 1)$ be the population of nonsmokers. Among the nonsmokers, a proportion $\beta \in (0, 1)$ do not patronize café 1. These are the radical nonsmokers. Regardless of prices, they patronize café 2 as long as its price is not higher than the reservation price.[4] The remaining nonsmokers $(1 - \beta)(1 - s)$ compare prices and choose the café with the lower price. If prices are equal, they patronize café 2. We assume that the population of smokers is smaller than the population of radical nonsmokers: $s < \beta(1 - s)$.[5] The cafés have the same marginal costs. For simplicity, we set them equal to zero. Café i, $i = 1, 2$, must charge the same price to all its customers. The cafés meet once and simultaneously choose prices p_i, $i = 1, 2$.

We can write the cafés' profits as follows:

$$\pi_1(p_1, p_2) = \begin{cases} p_1[s + (1 - \beta)(1 - s)] & \text{if } p_1 < p_2, \\ p_1 s & \text{if } p_1 \geq p_2,\ p_1 \leq r, \end{cases}$$

$$\pi_2(p_1, p_2) = \begin{cases} p_2(1 - s) & \text{if } p_2 \leq p_1, \\ p_2\beta(1 - s) & \text{if } p_2 > p_1,\ p_2 \leq r. \end{cases}$$

If café 1 is the lower-priced café, it attracts, besides the smokers s, the nonsmokers who compare prices $(1 - \beta)(1 - s)$. If not, it only attracts the smokers. If café 2 is the higher-priced café, only the radical nonsmokers $\beta(1 - s)$ will visit it; otherwise, it will appeal to all the nonsmokers $1 - s$.

We begin our analysis by showing that there is no Nash equilibrium in pure strategies. Intuitively, the existence of price-sensitive consumers makes undercutting the rival's price profitable, as in the traditional Bertrand model. However, the Bertrand outcome is not an equilibrium in this game, due to the consumers who do not compare prices.

Proposition 1: There is no Nash equilibrium in pure strategies in the price-setting game.

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3 Smokers' preferences are extreme in the sense that they only consume as long as they are allowed to smoke.

4 Like smokers, radical nonsmokers also have extreme preferences. They only consume if the environment is smoke-free.

5 The implications of this assumption will be discussed later.

Proof: Suppose there is an equilibrium in which both cafés choose the same price p, with $0 < p \leq r$. If café 1 deviates to $p - \varepsilon$ for a sufficiently small $\varepsilon > 0$, it will capture the nonradical nonsmokers and its profits will be larger. So, p cannot be an equilibrium. If $p = 0$ for both cafés, this unilateral deviation is impossible. However, deviations to a positive price will increase profits above zero.

Suppose that there is an equilibrium in which the cafés choose different prices. Without loss of generality, assume that $0 \leq p_2 < p_1 \leq r$. Then, café 2 has incentives to increase its price; its market share $(1 - s)$ does not then change but its profits are larger. □

However, the price-setting game has at least one equilibrium in mixed strategies. Dasgupta and Maskin (1986) showed the existence, under certain conditions, of equilibria in mixed strategies for discontinuous economic games that do not have pure-strategy equilibria. As applied to our game, these conditions require that the profits functions $\pi_i(p_i, p_j)$, $i = 1, 2$, be bounded and weakly lower semicontinuous and that $\pi_1 + \pi_2$ is continuous. It can easily be checked that these assumptions are satisfied in our model.

A mixed strategy for café i is a probability distribution $F_i(p)$ over the prices in the support b_i, $i = 1, 2$.[6] Let $\underline{b}_i$ and $\bar{b}_i$ be respectively the lower and upper bound of the support b_i, $i = 1, 2$. Let $f_i(p) = \mathrm{d}F_i(p)/\mathrm{d}p$ (almost everywhere) be the corresponding density function, which indicates the probability with which café i chooses p, $i = 1, 2$.

Let $\Pi_i(F_i(p), F_j(p))$ be café i's expected profits when its strategy is $F_i(p)$ and café j's strategy is $F_j(p)$, $i = 1, 2$, $i \neq j$. A Nash equilibrium in mixed strategies is a pair of probability distributions $(F_i^*(p), F_j^*(p))$ such that $\forall i = 1, 2$:

$$\Pi_i(F_i^*(p), F_j^*(p)) \geq \Pi_i(F_i(p), F_j^*(p)) \quad \forall F_i(p) \neq F_i^*(p),\ i \neq j\ .$$

Our problem is to find the equilibrium probability distributions $F_i^*(p)$, $i = 1, 2$, and their corresponding supports $b_i^* = [\underline{b}_i^*, \bar{b}_i^*]$. We specify a pair of probability distributions (one for each café) that constitute an equilibrium in Proposition 2 below; in the appendix, we prove that no other equilibrium exists.

6 Recall that the support of a distribution is the smallest closed set with probability one.

Proposition 2: Let $\beta(1-s) > s$. The following pair of probability distributions constitutes an equilibrium of the price-setting game.

$$F_1^*(p) = \begin{cases} 0 & \text{if } p < \underline{p}_2, \\ 1 - \dfrac{(r-p)\beta}{p(1-\beta)} & \text{if } \underline{p}_2 \leq p < r, \\ 1 & \text{if } p \geq r, \end{cases}$$

$$F_2^*(p) = \begin{cases} 0 & \text{if } p < \underline{p}_2, \\ 1 + \dfrac{s}{(1-\beta)(1-s)} - \dfrac{r\beta[1-\beta(1-s)]}{p(1-\beta)(1-s)} & \text{if } \underline{p}_2 \leq p < r, \\ 1 & \text{if } p \geq r, \end{cases}$$

where $\underline{p}_2 = \beta r$. Café 1's equilibrium expected profits are $\beta[1-\beta(1-s)]r$ and café 2's $\beta(1-s)r$.

Proof: First, we check that $F_1^*(p)$ and $F_2^*(p)$ are proper probability distributions. It is easy to see that for all $i = 1, 2$: (a) $F_i^*(\underline{p}_2) = 0$, (b) $\mathrm{d}F_i^*(p)/\mathrm{d}p > 0$, and (c) $F_i^*(r) = 1$. Given $F_1^*(p)$ and $F_2^*(p)$, we can write cafés 1 and 2's expected profits when they play those strategies as follows:

$$\begin{aligned} \Pi_1(F_1^*(p), F_2^*(p)) &= pF_2^*(p)s + p(1 - F_2^*(p))[1-\beta(1-s)] \\ &= \beta[1-\beta(1-s)]r \\ \Pi_2(F_1^*(p), F_2^*(p)) &= pF_1^*(p)\beta(1-s) + p(1-F_1^*(p))(1-s) \\ &= p(1-s) - p(1-s)(1-\beta)\left[1 - \frac{(r-p)\beta}{p(1-\beta)}\right] \\ &= \beta(1-s)r\ . \end{aligned}$$

Let $F_i(p), i = 1, 2$, be a probability distribution with support $[\underline{p}_2, r]$ and $F_i(p) \neq F_i^*(p)$. Then, $\Pi_i(F_i(p), F_j^*(p)) = \Pi_i(F_i^*(p), F_j^*(p)), i, j = 1, 2, i \neq j$. For $p \in [0, \underline{p}_2)$, café 2's expected profits when it plays p and café 1 plays $F_1^*(p)$ are:

$$\begin{aligned} \Pi_2(F_1^*(p), p) &= p(1-s) < \underline{p}_2(1-s) = \beta(1-s)r \\ &= \Pi_2(F_1^*(p), F_2^*(p))\ . \end{aligned}$$

Similarly, café 1's expected profits when it plays p and café 2 plays

$F_2^*(p)$ are:

$$\Pi_1(p, F_2^*(p)) = p[1 - \beta(1-s)] < \underline{p}_2[1 - \beta(1-s)] \\ = \Pi_1(F_1^*(p), F_2^*(p)) \, .$$

When $p > r$, we have that $\Pi_i(p, F_j^*(p)) = 0$, $i = 1, 2$, $i \neq j$. For $p = r$, we have that $\Pi_2(F_1^*(p), r) = \beta(1-s)r$ because café 2 is the higher-priced café with probability one.

Let $f_2^*(r)$ be the probability that café 2 chooses r. We can write $f_2^*(r)$ as follows:

$$f_2^*(r) = 1 - \text{Prob}(p_2 < r) = \frac{\beta[1 - \beta(1-s)]}{(1-s)(1-\beta)} - \frac{s}{(1-s)(1-\beta)} \\ = \frac{\beta(1-s) - s}{1-s} \, .$$

When café 1 plays the reservation price r, there is a positive probability of a tie because café 2 is playing r with a strictly positive probability. By taking into account how the price-sensitive consumers choose cafés when prices are equal, we have $\Pi_1(r, F_2^*(p)) = (1 - f_2^*(r))rs + f_2^*(r)rs = rs$ and $\Pi_1(F_1^*(p), F_2^*(p)) - \Pi_1(r, F_2^*(p)) = (1-\beta)r[\beta(1-s) - s] > 0$. Thus, the probability distributions $(F_1^*(p), F_2^*(p))$ are an equilibrium. □

The equilibrium probability distributions for the duopoly case, although they have a common support, are different. The equilibrium probability distribution of the café with the larger loyal population (café 2) stochastically dominates that of café 1. Café 2 has a higher probability of charging higher prices than café 1. While smokers enjoy a positive surplus with probability one, there is a positive probability that café 2 exploits the radical nonsmokers by choosing the monopoly price r. Note that café 2's equilibrium expected profits are equal to those it would obtain if it charged the monopoly price, while café 1's are like those it would obtain if it charged the lower bound of the support of the equilibrium probability distributions.

The analysis is qualitatively similar when the population of smokers is larger than that of radical nonsmokers [$s > \beta(1-s)$], although the equilibrium probability distributions are different. In this case, the common support is $[\underline{p}_1, r]$, where $\underline{p}_1 = s/[1 - \beta(1-s)]$, and café 1 chooses, in equilibrium, the monopoly price r with positive probability. Café 1 and 2's equilibrium expected profits are, respectively, rs and $[rs(1-s)]/[1 - \beta \cdot (1-s)]$. When $\beta(1-s) = s$, both cafés choose an identical probability distribution. No café chooses the monopoly price with positive probability, and in equilibrium, their expected profits are $r\beta(1-s)$.

So far, we have taken the smoking policies as exogenously given. When we add a previous stage to the price-setting game where the cafés select their smoking policies, it can be shown that asymmetric policies arise in equilibrium. When smoking policies are endogenous, we need to specify how consumers decide when both cafés choose the same policy. If consumers compare prices and select the cheaper café, then, under a common policy, we would have the Bertrand result. Thus, cafés differentiate themselves as much as they can to soften competition and asymmetric policies arise as an equilibrium outcome.

3 The Model with Three Cafés

In this section, we extend the model by introducing a third player. We assume that a duopolistic market structure with asymmetric policies as described in Sect. 2 has been in place for some time and a new café (café 3) enters the market. In particular, we continue to assume that café 1 allows smoking while café 2 is smoke-free. Due to repeated visits, smokers become loyal to café 1. Radical nonsmokers have been patronizing café 2 and also become loyal to it. By taking into account the equilibrium strategies in the duopoly case, there is no guarantee that the same café is the lower-priced café every time. Therefore, the nonradical nonsmokers do not patronize the same café every time, and they do not develop loyalty.[7] Given loyalty, the smoking policy adoped by the new café is irrelevant. Its potential consumers are those who compare prices and, for them, the smoking policy does not play any role in their decisions. Without loss of generality, we assume that the new café follows a smoke-free policy. If cafés 2 and 3 charge the lower price, a proportion $d_j \in (0, 1)$ of the population of consumers who compare prices go to café j, $j = 2, 3$ and $d_2 + d_3 = 1$. We can write the cafés' profits as:

7 Assuming perfect loyalty simplifies the notation without changing qualitatively the results. A similar analysis can be made if we assume that only a fraction of smokers and radical nonsmokers become loyal instead of all of them. The same applies to the population of nonsmokers who compare prices.

$$\pi_1(p_1, p_2, p_3) = \begin{cases} p_1[s + (1-\beta)(1-s)] & \text{if } p_1 < \min(p_2, p_3), \\ p_1 s & \text{if } p_1 \geq \min(p_2, p_3),\ p_1 \leq r, \end{cases}$$

$$\pi_2(p_1, p_2, p_3) = \begin{cases} p_2(1-s) & \text{if } \begin{cases} p_2 \leq p_1 < p_3, \\ p_2 < p_3 \leq p_1, \end{cases} \\ p_2[\beta(1-s) \\ \quad + d_2(1-\beta)(1-s)] & \text{if } p_2 = p_3 \leq p_1, \\ p_2\beta(1-s) & \text{if } \begin{cases} p_2 > p_3, \\ p_1 < p_2 < p_3,\ p_2 \leq r, \end{cases} \end{cases} \tag{1}$$

$$\pi_3(p_1, p_2, p_3) = \begin{cases} p_3(1-s)(1-\beta) & \text{if } \begin{cases} p_3 \leq p_1 < p_2, \\ p_3 < p_2 \leq p_1, \end{cases} \\ p_3 d_3(1-\beta)(1-s) & \text{if } p_3 = p_2 \leq p_1, \\ 0 & \text{if } \begin{cases} p_3 > p_2, \\ p_1 < p_3 < p_2,\ p_3 \leq r. \end{cases} \end{cases}$$

When café 1 charges the lowest price, the population of smokers and price-sensitive nonsmokers patronize it. When its price is the highest, it only sells to the population of smokers. The same considerations lead to the profit function for café j, $j = 2, 3$ in (1). In particular, if café 3 is the highest-priced one, no consumer will patronize it.

The price-setting game here, as in Sect. 2, has no Nash equilibrium in pure strategies. The introduction of an additional player changes the qualitative features of the equilibrium (a). When we have two players, both of them care about the price-sensitive consumers and compete for them. With three or more players, the attractiveness of capturing the price-sensitive consumers diminishes and some players consider focusing on exploiting their loyal consumers to be more attractive. There is, at least, one hybrid equilibrium in which the café with the larger loyal population chooses the monopoly price r and the other two cafés play a mixed strategy.[8] Before

8 Although there may exist equilibria in which all the players use mixed strategies, we have been able to find them. Here, we only present those equilibria in which one of the players plays a pure strategy while the remaining players use a mixed one.

characterizing the hybrid equilibrium, we must find two probability distributions that would constitute the unique equilibrium in the duopoly case when only cafés 2 and 3 are in the market and their profit functions are like those in (1). The analysis in the last section and in the appendix can be carried over, with slight modifications due to the different specifications of the profit functions, to obtain the following equilibrium probability distributions:

$$G_2^*(p) = \begin{cases} 0 & \text{if } p < \underline{p}_2, \\ 1 - \dfrac{r\beta}{p} & \text{if } \underline{p}_2 \le p < r, \\ 1 & \text{if } p \ge r, \end{cases}$$

$$G_3^*(p) = \begin{cases} 0 & \text{if } p < \underline{p}_2, \\ 1 - \dfrac{(r-p)\beta}{p(1-\beta)} & \text{if } \underline{p}_2 \le p < r, \\ 1 & \text{if } p \ge r, \end{cases}$$

where $\underline{p}_2 = r\beta$. The Proposition below characterizes a hybrid equilibrium for the price-setting game when we have three cafés.

Proposition 3: Let $\beta(1-s) \le s$. The strategy profile $\{r, G_2^*(p), G_3^*(p)\}$ is an equilibrium of the price-setting game. In equilibrium, café 1's expected profits are rs, café 2's $r\beta(1-s)$, and café 3's $r\beta(1-s)(1-\beta)$.

Proof: Given $\{r, G_2^*(p), G_3^*(p)\}$, café 1's expected profits are rs because café 3's price is smaller than r with probability one. Thus, café 1 only sells to the population of smokers. We can write café 2 and 3's expected profits as follows:

$$\begin{aligned} \pi_2(r, G_2^*(p), G_3^*(p)) &= p\beta(1-s)G_3^*(p) + p(1-s)(1-G_3^*(p)) \\ &= r\beta(1-s)\,, \\ \pi_3(r, G_2^*(p), G_3^*(p)) &= p(1-\beta)(1-s)(1-G_2^*(p)) \\ &= \underline{p}_2(1-\beta)(1-s)\,. \end{aligned}$$

Given that café 1 chooses r and café 3 chooses $G_3^*(p)$, there is no deviation that gives café 2 expected profits greater than $r\beta(1-s)$. Note that café 2's expected profits are equal to those obtained if it focuses only on its loyal consumers and charges them the monopoly price. As café 1 sells only to its loyal customers, there is no profitable deviation for café 2.

Similarly, given that café 1 chooses r and café 2 $G_2^*(p)$, there is no profitable deviation for café 3. As the smokers only buy from café 1, the problem for cafés 2 and 3 is similar to that when we have only 2 cafés. Thus, $G_3^*(p)$ is café 3's best response to $G_2^*(p)$ and r.

We must now see if there is a profitable deviation for café 1 when cafés 2 and 3 play, respectively, $G_2^*(p)$ and $G_3^*(p)$. Given these strategies, café 1's expected profits for $p \in [\underline{p}_2, r]$ are:

$$\begin{aligned}\Pi_1(p, G_2^*(p), G_3^*(p)) &= ps + p(1-s)(1-\beta)(1-G_2^*(p))(1-G_3^*(p)) \\ &= ps + \frac{r\beta^2(r-p)(1-s)}{p}\,. \qquad (2)\end{aligned}$$

By taking into account the expected profits in (2), we can write:

$$\frac{\mathrm{d}\Pi_1(p, G_2^*(p), G_3^*(p))}{\mathrm{d}p} = s - \frac{r^2\beta^2(1-s)}{p^2}\,,$$
$$\frac{\mathrm{d}^2\Pi_1(p, G_2^*(p), G_3^*(p))}{\mathrm{d}p^2} > 0\,.$$

Thus, $\tilde{p} = r\beta\sqrt{(1-s)/s} = \arg\min \Pi_1(p, G_2^*(p), G_3^*(p))$. For $s \leq 0.5$, $r > \tilde{p} \geq r\beta$. We also have that $[\mathrm{d}\Pi_1(p, G_2^*(p), G_3^*(p))/\mathrm{d}p]\big|_{p=r} = s - \beta^2(1-s) > 0$, given that $s \geq \beta(1-s)$. It follows that café 1's expected profits are maximized at $p = r$. For $s > 0.5$, $\tilde{p} < r\beta < \underline{p}_1$, where $\underline{p}_1 = rs/(s + (1-s)(1-\beta))$ is the minimum price café 1 is willing to choose. Thus, $\forall p \in [\underline{p}_1, r]$, café 1's expected profits increase with p. Thus, its profits are maximized at $p = r$. □

In the above equilibrium, the café with the largest loyal population (café 1) exploits its loyal consumers by choosing the monopoly price and the other two cafés compete for the price-sensitive consumers. Under duopoly, both cafés care about the price-sensitive consumers. The analysis suggests that when there are $n > 3$ cafés, the biggest $n - 2$ ones focus on their loyal consumers and the two cafés with the smallest loyal populations compete for the price-sensitive consumers.

The strategy profile in Proposition 3 fails to be an equilibrium if $s < \beta \cdot (1-s)$. In this case, $\underline{p}_1 < \underline{p}_2$ and café 1 can increase its profits by deviating to any price $p \in (\underline{p}_1, \underline{p}_2)$. Note that café 1's price would certainly be the lowest one. As $\Pi_1(\underline{p}_1, G_2^*(p), G_3^*(p)) = \Pi_1(r, G_2^*(p), G_3^*(p)) = rs$, it follows that $\Pi_1(p, G_2^*(p), G_3^*(p)) > rs$ for $p \in (\underline{p}_1, \underline{p}_2)$. However, when $s < \beta(1-s)$, there also exists a hybrid equilibrium in which café 2 plays a pure strategy. Consider the following probability distributions:

$$H_1^*(p) = \begin{cases} 0 & \text{if } p < \underline{p}_1, \\ 1 - \dfrac{rs}{p[s + (1-s)(1-\beta)]} & \text{if } \underline{p}_1 \le p < r, \\ 1 & \text{if } p \ge r, \end{cases}$$

$$H_3^*(p) = \begin{cases} 0 & \text{if } p < \underline{p}_1, \\ 1 - \dfrac{(r-p)s}{p(1-\beta)(1-s)} & \text{if } \underline{p}_1 \le p < r, \\ 1 & \text{if } p \ge r, \end{cases}$$

with $\underline{p}_1 = s/(s + (1-s)(1-\beta))$, that constitute the unique equilibrium when only cafés 1 and 3 are in the market. The following Proposition, stated without proof, characterizes a hybrid equilibrium.[9]

Proposition 4: Let $\beta(1-s) > s$. The strategy profile $\{H_1^*(p), r, H_3^*(p)\}$ is an equilibrium of the price-setting game. In equilibrium, café 1's expected profits are rs, café 2's $r\beta(1-s)$, and café 3's $r\underline{p}_1(1-\beta)(1-s)$.

4 Conclusions

We have considered a model of horizontal product differentiation in which the population of consumers is segmented. Segmentation is based on smoking habits and attitudes toward comparing prices. In the duopoly case, both cafés compete for the price-sensitive consumers and the unique equilibrium of the price-setting game is in mixed strategies; however, the nature of the equilibria change when we introduce more than two cafés. We have found an equilibrium in which one café plays a pure strategy (the monopoly price) and the remaining cafés play a mixed strategy. The café with the biggest loyal population exploits it when there are more than two cafés, leaving the price-sensitive consumers to the others.

Although the model has been used to analyze price competition in a specific context, its validity is more general. The main result is also applicable to markets in which new firms, in order to capture market share, compete aggressively to attract those consumers who are price-sensitive, while the settled firms focus only on their customer bases and charge them higher prices.

9 The proof is like that of Proposition 3.

Appendix

In this appendix, we show that the price-setting game in Sect. 2 has a unique equilibrium in mixed strategies and that the equilibrium probability distributions are those given in Proposition 2. Our problem is to characterize the equilibrium probability distributions $F_i^*(p)$, $i = 1, 2$ and their corresponding supports $b_i^* = [\underline{b}_i^*, \bar{b}_i^*]$. We proceed by, first, establishing the properties that a mixed strategy must satisfy to be part of an equilibrium. Once that is done, we find the equilibrium of the price-setting game.

Lemma 1: For $i = 1, 2$, $b_i^* \subseteq [\underline{p}_i, r]$, where $\underline{p}_1 = rs/(1-\beta(1-s))$ and $\underline{p}_2 = r\beta$, $i = 1, 2$.

Proof: As consumers do not buy if the price is higher than the reservation price r, no café will choose a price $p > r$. Café 1 can guarantee a profit of at least rs in equilibrium by choosing a price $p_1 = r$. Let $\underline{p}_1$ be the lowest price that café 1 is willing to choose. When café 1 chooses $\underline{p}_1$ and it is the lowest-priced café, its profits must be equal to rs, which is the profit when it chooses r and it is the highest-priced café:

$$rs = \underline{p}_1[1 - \beta(1-s)] .$$

It follows that $\underline{p}_1 = rs/(1-\beta(1-s))$.

Similarly, café 2 can guarantee a profit of at least $\beta(1-s)r$ in equilibrium by choosing a price $\underline{p}_2 = r$. Let $\underline{p}_2$ be the lowest price that café 2 is willing to choose. When café 2 chooses $\underline{p}_2$ and it is the lowest-priced café, its profits must be equal to $\beta(1-s)r$, which is the profit when it chooses r and it is the highest-priced café:

$$\beta(1-s)r = \underline{p}_2(1-s) .$$

It follows that $\underline{p}_2 = \beta r$. □

The next step is to rule out the presence of any discontinuities in the equilibrium probability distributions at prices $p_i \in [\underline{b}_i^*, \bar{b}_i^*)$, $i = 1, 2$. Intuitively, if café i's equilibrium density function $f_i^*(p)$ has a mass point at $p_i \in [\underline{b}_i^*, \bar{b}_i^*)$ and $p_i < p_j$, café i can increase its expected profits by deviating and choosing $p_i + \varepsilon$ with the same probability with which it was choosing p_i, and p_i with probability zero.

If $p_i = p_j$, café i can increase its expected profits by deviating and choosing $p_i - \varepsilon$ with the same probability with which it was choosing p_i, and p_i with probability zero. It will lose profits of order ε, but it will gain all the price-sensitive consumers when café j chooses p_j. In equilibrium, probability distributions have, at most, a discontinuity at $\bar{b}_i^*$, $i = 1, 2$. We formalize this argument in the following lemma.

Lemma 2: Let $(F_1^*(p), F_2^*(p))$ be an equilibrium of the price-setting game. Let $b_i^* = [\underline{b}_i^*, \bar{b}_i^*] \subseteq [\underline{p}_i, r]$ be the corresponding supports, $i = 1, 2$. Then, no distribution $F_i^*(p)$ has a point of positive probability at $p_i \in [\underline{b}_i^*, \bar{b}_i^*)$, $i = 1, 2$.

Proof: Suppose that $p_i \in [\underline{b}_i^*, \bar{b}_i^*)$, $i = 1, 2$ is a mass point of the probability density function $f_i^*(p)$, $i = 1, 2$. Without loss of generality, let $p_2 < p_1$. Café 2's expected profits in equilibrium are given by:

$$\Pi_2(F_1^*(p), F_2^*(p)) = p_2 F_1^*(p_2)\beta(1 - s) + p_2(1 - F_1^*(p_2))(1 - s) .$$

Consider the following deviation by café 2: choose $p_2 + \varepsilon$, for a small $\varepsilon > 0$, with probability $f_2^*(p_2)$ and p_2 with probability zero. Café 2's expected profits from this deviation are:

$$\begin{aligned}\Pi_1(F_1^*(p), p_2 + \varepsilon) &= (1 - s)(p_2 + \varepsilon)(1 + F_1^*(p_2 + \varepsilon)\beta - F_1^*(p_2 + \varepsilon)) \\ &= (1 - s)[(p_2 + \varepsilon)F_1^*(p_2)\beta \\ &\quad + (p_2 + \varepsilon)(1 - F_1^*(p_2))] \\ &> \Pi_2(F_1^*(p), F_2^*(p)) .\end{aligned}$$

Let $p_1 = p_2$. Consider the following deviation by café 2: choose $p_2 - \varepsilon$, for a small $\varepsilon > 0$, with probability $f_2^*(p_2)$ and p_2 with probability zero. Café 2's expected profits from this deviation are:

$$\begin{aligned}\Pi_1(F_1^*(p), p_2 - \varepsilon) &= (1 - s)(p_2 - \varepsilon)(1 + F_1^*(p_2 - \varepsilon)\beta - F_1^*(p_2 - \varepsilon)) \\ &= (p_2 - \varepsilon)[F_1^*(p_2) - f_1^*(p_1)]\beta(1 - s) \\ &\quad + (p_2 - \varepsilon)[1 - F_1^*(p_2) + f_1^*(p_2)](1 - s) .\end{aligned}$$

When $\varepsilon \to 0$, $\Pi_2(F_1^*(p), p_2 - \varepsilon) \to \Pi_2(F_1^*(p), F_2^*(p)) + p_2 f_1^*(p_2) \cdot (1 - s)(1 - \beta) > \Pi_2(F_1^*(p), F_2^*(p))$. Thus, café 2's expected profits increase, which contradicts the assumption that $F_2^*(p)$ is an equilibrium probability distribution.[10]

Suppose now that only one distribution, say café i's, has a point of positive probability at $p_i \in [\underline{b}_i^*, \bar{b}_i^*)$. Café i can deviate and choose $p_i + \varepsilon$

10 The argument carries over when we have more than one mass point.

with probability $f_i^*(p_i)$ and p_i with probability zero. It follows from the above reasoning that café i can increase its expected profits with this deviation. □

Since there are no mass points in the equilibrium density functions for prices $p_i \in [\underline{b}_i^*, \bar{b}_i^*)$, $i = 1, 2$, the equilibrium probability distributions are continuous functions on $(\underline{b}_i^*, \bar{b}_i^*)$, $i = 1, 2$. We can use Lemma 1 to further characterize the supports b_i^* of the equilibrium probability distributions, $i = 1, 2$.

Lemma 3: The equilibrium probability distributions $F_i^*(p)$ have a common support $b^* = [\underline{b}^*, \bar{b}^*] \subseteq [\underline{p}_2, r]$, $i = 1, 2$.

Proof: From Lemma 1, we know that $b_2^* \subseteq [\underline{p}_2, r]$. Suppose that $\underline{b}_1^* < \underline{p}_2$. Then, $\forall p \in b_1^*$ with $p < \underline{p}_2$, café 1's profits increase with p, because café 1 is certainly the lowest-priced café. But, then, $F_1^*(p)$ cannot be an equilibrium probability distribution because, in equilibrium, expected profits must be equal for all the prices in the distribution support. Thus, $\underline{b}_1^* \geq \underline{p}_2$ and $\forall i = 1, 2$, $b_i^* \subseteq [\underline{p}_2, r]$.

Suppose that $\underline{b}_i^* > \underline{b}_j^*$, $i = 1, 2$, $i \neq j$. Then, $\forall p \in b_j^*$ with $p < \underline{b}_i^*$, café j would certainly be the lowest-priced café and its expected profits would increase with p, which violates the fact that $F_j^*(p)$ is an equilibrium probability distribution. Thus, $\underline{b}_i^* \leq \underline{b}_j^*$. But, $\underline{b}_i^*$ cannot be smaller than $\underline{b}_j^*$ because café i's profits would increase with p for $p \in b_i^*$ with $p < \underline{b}_j^*$. Thus, $\underline{b}_i^* = \underline{b}_j^*$, $i = 1, 2$, $i \neq j$.

Suppose that $\bar{b}_i^* > \bar{b}_j^*$, $i = 1, 2$, $i \neq j$. Then, $\forall p \in b_i^*$ with $p > \underline{b}_j^*$, café i would certainly be the highest-priced café and its expected profits would increase with p, which violates the fact that $F_i^*(p)$ is an equilibrium probability distribution. Thus, $\bar{b}_i^* \leq \bar{b}_j^*$. But, $\bar{b}_i^*$ cannot be smaller than $\bar{b}_j^*$ because café i's profits would increase with p for $p \in b_i^*$ with $p < \bar{b}_j^*$. Thus, $\bar{b}_i^* = \bar{b}_j^*$, $i = 1, 2$, $i \neq j$. □

From Lemma 2, we know that the equilibrium distributions have, at most, a discontinuity at $\bar{b}^*$. The following two lemmas show that only one equilibrium distribution can have a discontinuity at $\bar{b}^*$.

Lemma 4: Let $(F_1^*(p), F_2^*(p))$ be an equilibrium of the price-setting game. Let $b^* = [\underline{b}^*, \bar{b}^*] \subseteq [\underline{p}_2, r]$ be the common support. Then, both distributions cannot have a point of positive probability at $\bar{b}^*$.

Proof: Suppose that $\bar{b}^*$ is a mass point of the equilibrium density functions $f_i^*(p)$, $i = 1, 2$. There is then a positive probability of a tie at $\bar{b}^*$. As the number of points with positive mass in any probability distribution is countable, we can find an arbitrarily small ε such that $\bar{b}^* - \varepsilon$ is chosen with probability zero.

Consider the following deviation by café 1: choose $\bar{b}^* - \varepsilon$ with probability $f_1^*(\bar{b}^*)$ and $\bar{b}^*$ with probability zero. The increase in expected profits for café 1 is given by:

$$(\bar{b}^* - \varepsilon) f_1^*(\bar{b}^*) f_2^*(\bar{b}^*)[1 - \beta(1 - s)] - \bar{b}^* f_1^*(\bar{b}^*) f_2^*(\bar{b}^*) s$$

and for a small ε, it is positive. But $(F_1^*(p), F_2^*(p))$ was an equilibrium. So, we have a contradiction. □

Lemma 5: Let $(F_1^*(p), F_2^*(p))$ be an equilibrium of the price-setting game. Let $b^* = [\underline{b}^*, \bar{b}^*] \subseteq [\underline{p}_2, r]$ be the common support. Then, it is not possible for both cafés not to choose the upper bound of the support $\underline{b}^*$.

Proof: Suppose that both cafés do not choose the upper bound of the support $\bar{b}^*$. Café 2's expected profits in equilibrium are:

$$\begin{aligned}\Pi_2(F_1^*(p), F_2^*(p)) = {} & pF_1^*(p)\beta(1 - s) \\ & + p(1 - F_1^*(p))(1 - s) \quad \forall p \in (\underline{b}^*, \bar{b}^*) .\end{aligned}$$

Consider a sequence of prices (p^2) with $f_1^*(p^2) > 0$ such that $(p^2) \to \bar{b}^*$. When $p^2 \to \bar{b}^*$, $F_1^*(p^1) \to 1$ and $\Pi_2(F_1^*(p), F_2^*(p)) \to p^2\beta \cdot (1 - s)$.

If café 2 deviates and chooses $\bar{b}^*$, its profits are $\bar{b}^*\beta(1-s) > p^2\beta(1-s)$. Thus, it is not possible for both equilibrium distributions not to have a discontinuity at $\bar{b}^*$. □

From Lemmas 4 and 5, it follows that in equilibrium, one café's density function must have a mass point at $\bar{b}^*$. It turns out that café 2 chooses the upper bound of the support $\bar{b}^*$, with positive probability.

Lemma 6: Café 2's equilibrium probability distribution has a point of positive probability at $\bar{b}^*$: $F_2^*(\bar{b}^*) < 1$.

Proof: Suppose that $F_1^*(\bar{b}^*) < 1$ and $F_2^*(\bar{b}^*) = 1$. Then, when café 1 chooses $\bar{b}^*$, it is certainly undercut. Thus, in equilibrium, café 1's expected profits are $\bar{b}^* s \leq rs$. If café 1 deviates from $F_1^*(p)$ and chooses $\underline{p}_2$ with probability one, it will certainly be the lowest-price café because, from Lemma 2, café 2 chooses $\underline{b}^* \geq \underline{p}_2$ with zero probability. Thus, café 1's profits are $\beta[1 - \beta(1 - s)]r$. This deviation is profitable if:

$$\beta[1 - \beta(1 - s)]r > \bar{b}^* s\ .$$

It suffices to show that this condition is satisfied when $\bar{b}^* = r$. Suppose it is not satisfied. Then, we have

$$\beta[1 - \beta(1 - s)]r \leq rs\ ,$$

which is a contradiction given $\beta(1-s) > s$. Thus, $F_1^*(\bar{b}^*) = 1$ and $F_2^*(\bar{b}^*) < 1$. □

From Lemma 6, it follows that in equilibrium, café 2's expected profits are $\bar{b}^* \beta(1-s)$. We can determine the upper bound of the equilibrium probability distributions by noticing that $\bar{b}^* \beta(1 - s) \geq r\beta(1 - s)$ and $\bar{b}^* \leq r$. Expected profits in equilibrium must be, at least, as large as the minimax profit level $r\beta(1-s)$. Thus, the upper bound of the common support of the equilibrium probability distributions is the reservation price r, and café 2's expected profits in equilibrium are $r\beta(1 - s)$. In equilibrium, café 1's expected profits are equal to those it will get if it focuses only on its loyal consumers and charges them the monopoly price.

The lower bound of the support of the equilibrium probability distributions must be $\underline{p}_2$. If $\underline{b}^* > \underline{p}_2$, café 2 can deviate from $F_1^*(p)$ and choose $\underline{b}^*$. It will certainly be the lowest-priced café and its profits will be $\underline{b}^*(1 - s) > r\beta(1 - s)$, which contradicts the assumption that $F_2^*(p)$ is the equilibrium probability distribution for café 2.

Finally, note that café 1 must assign positive probability to any interval containing $\underline{p}_2$. Otherwise, café 2 will find profitable any deviation to $\underline{p}_2 + \varepsilon$ for $\varepsilon > 0$. Thus, it follows that, in equilibrium, café 1's expected profits must be equal to $\underline{p}_2[1 - \beta(1 - s)]$.

It only remains to characterize the distributions $F_1^*(p)$ and $F_2^*(p)$. It turns out that there is a unique pair of probability distributions satisfying all the properties derived in the above lemmas. In equilibrium, all the prices in the distribution supports must provide the same expected profits. Given that café 2's expected profits are $r\beta(1-s)$ and café 1's are $\underline{p}_2[1-\beta(1-s)]$, we can write:

$$r\beta(1-s) = pF_1^*(p)\beta(1-s) + p(1-F_1^*(p))(1-s) \,,$$
$$\underline{p}_2[1-\beta(1-s)] = pF_2^*(p)s + p(1-F_2^*(p))[1-\beta(1-s)] \,,$$

and solving for $F_1^*(p)$ and $F_2^*(p)$ we get the distributions given in Proposition 2.

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