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ROBUST SPECIFICATION INFLUENCE ON FEEDBACK CONTROL STRATEGIES

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Abstract: The feedback control structures are required in the presence of any kind of uncertainty. The benefits of the feedback are mainly paid with an excessive bandwidth that amplifies the sensor noise, saturating the actuators. This implies the necessity of design trade-offs highly transparent from the Quantitative Feedback Theory. Supposing a given uncertainty for a plant, the set of performance and stability requirements will condition the problem solution. Taking advantage of the QFT bound typology and formulation, the contribution of the robust specification values to their simultaneous achievement will be analysed and particularly as regards the challenging task of the controller design. *Copyright © 2002 IFAC*

Keywords: Robust control, Uncertain dynamical systems, Quantitative Feedback Theory.

1. INTRODUCTION

The feedback control is justified just in the presence of model and/or disturbance uncertainties (Horowitz, 1963). In this case, the benefits of the feedback inevitably have a price. The design trade-offs in a minimum phase SISO feedback control system requires, in terms of the open loop transmission $L=GP$: (*a*) a low-medium frequency gain $|L|$ high enough for an appropriate robust performance in the reference signal tracking and the disturbance rejection tasks; (*b*) a $|L|$ decrement as fast as possible with frequency that avoids a large noise amplification mainly at the actuator inputs (Horowitz, 1973); (*c*) a constraint of the $|L|$ decrement rate by the robust stability requirements.

The feedback control trade-offs are expected to become tougher with an increasing uncertainty size and more ambitious specifications. Supposing a fixed uncertainty, this work is aimed at analyzing the contribution of the specification values to the feedback trade-offs. A quantitative formulation of the problem, such as the Quantitative Feedback Theory (QFT) (Horowitz, 1963) is very helpful in this sense. To be precise, the QFT bounds contain all the information. They stand for the set of control specifications and the plant uncertainty in the Nichols Chart (NC). Therefore, a thorough study of them answers two crucial questions in terms of control theory: the compatibility amongst different robust specifications and the existence of a controller capable of meeting simultaneously all of them.

Several dissertations on the previous questions can be found in: Åstrom (2000a,b), Skogestad and Postlethwaite (1996). The QFT has already offered some answers. Horowitz (1979, App.1) proved the existence of a unique optimum L that satisfies a number of robust specifications, under certain constraints on the plant and on the specifications. Gil-Martínez and García-Sanz (2001) dealt with the simultaneous meeting of the control specifications without those restrictions. Three bound typologies were identified for general feedback purposes and their compatibility was discussed. On the basis of this previous work, the present paper studies more in detail the tolerance contribution to the bound compatibility and, mainly, to the difficulties in the loop shaping step (controller design). Chait and Yaniv (1993) developed bound quadratic inequalities as an aid in the automatic bound computation and the controller design. This bound formulation is used in the present work.

Section 2 covers the general feedback control requirements and includes a quick overview of previous contributions to the bound formulation and classification. Section 3 reviews the bound formulas to identify the influence of each robust specification model on its representative bound. Section 4 is devoted to the difficulties related to the bound compatibility and, mainly, to the feedback controller design as a consequence of the more ambitious specifications.

2. FEEDBACK CONTROL FROM QFT

2.1 Multiple robust control specifications

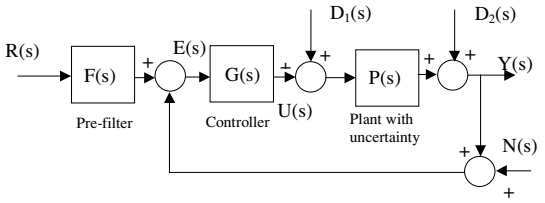

Figure 1: Control structure.

Supposing an uncertain plant, $\{P\}$, and unmeasurable disturbances, $\{D_1, D_2\}$, Figure 1 shows a general two-degrees-of-freedom (Horowitz, 1963) feedback control structure. The controller $G(s)$ must guarantee a robust stability, reduce the closed loop uncertainty and attenuate the input and output disturbances. $G(s)$ also ensures a high frequency gain low enough to minimize the 'cost of the feedback' (excessive bandwidth) (Bode, 1945; Horowitz, 1973). Afterwards, the prefilter $F(s)$ shapes the output performance in the reference tracking. Unity feedback is assumed for simplicity.

The QFT outlines the control objectives in the frequency domain and in terms of inequalities (Houpis and Rassmussen, 1999; Yaniv, 1999). Closed loop specification tolerances $_k$ are imposed on the system's transfer functions $|T_k|$ from some inputs to some outputs (see $|T_k|$ $_k$, k=1,...,5 in Table 1). $_{k=1}$ restricts the complementary sensitivity function $|T|=|L/1+L|$, $L=GP$, implying conditions on the robust stability, the robust control effort in the input disturbance rejection ($|U/D_1|$) and the robust sensor noise attenuation ($|Y/N|$). $_{k=2}$ and $_{k=3}$ constrain the sensitivity functions $|S|=|1/1+L|$ and $|S|=|P/1+L|$, respectively, for a robust rejection of the system output ($|Y/D_2|$) and input ($|Y/D_1|$) disturbances. $_{k=4}$ restricts the robust control effort $|G/1+L|$ for the system output disturbance rejection ($|U/D_2|$), the noise attenuation ($|U/N|$) and the tracking of the reference signals ($|U/RF|$). The upper $_{5\text{sup}}$ and lower $_{5\text{inf}}$ models constrain the signal tracking.

Table 1: Feedback control specifications

Transfer functions and specification models	Eq.
$\|T_1(j\ )\|=\left\|\frac{L(j\ )}{1+L(j\ )}\right\|=\left\|\frac{U(j\ )}{D_1(j\ )}\right\|=\left\|\frac{Y(j\ )}{N(j\ )}\right\|$ $_1(\ ),$ $_1$	(1)
$\|T_2(j\ )\|=\left\|\frac{1}{1+L(j\ )}\right\|=\left\|\frac{Y(j\ )}{D_2(j\ )}\right\|$ $_2(\ ),$ $_2$	(2)
$\|T_3(j\ )\|=\left\|\frac{P(j\ )}{1+L(j\ )}\right\|=\left\|\frac{Y(j\ )}{D_1(j\ )}\right\|$ $_3(\ ),$ $_3$	(3)
$\|T_4(j\ )\|=\left\|\frac{G(j\ )}{1+L(j\ )}\right\|=\left\|\frac{U(j\ )}{D_2(j\ )}\right\|=\left\|\frac{U(j\ )}{N(j\ )}\right\|=\left\|\frac{U(j\ )}{R(j\ )F(j\ )}\right\|$ $_4(\ ),$ $_4$	(4)
$_{5\text{inf}}(\ )$ $\|T_5(j\ )\|=\left\|F(j\ )\frac{L(j\ )}{1+L(j\ )}\right\|=\left\|\frac{Y(j\ )}{R(j\ )}\right\|$ $_{5\text{sup}}(\ ),$ $_5$	(5)

2.2 QFT bound formulation and typologies

Considering a particular frequency vector $\{\ _i\}$, the QFT translates the closed-loop specifications $\{\ _k(\ _i)\}$ and the plant *templates* (boundaries of the uncertain LTI plant at each frequency in $\{\ _i\}$, on the NC; Chen and Ballance, 1999) into bounds on the controller $G(j\ _i)$ or on the nominal loop transmission $L_0=P_0G$ (Houpis and Rasmussen, 1999; Yaniv, 1999). Chait and Yaniv (1993) developed an iterative algorithm to compute the bounds as follows. Borghesani *et al.*, 1994 implemented it in their software package. Each plant in the $_i$-template can be expressed in its polar form as $P(j\ _i)=p\ e^{j\ }=p\ $ and, likewise, the controller polar form is $G(j\ _i)=g\ e^{j\ }=g\ $. Then, substituting and rearranging the inequalities (1) to (5) in Table 1, they are reduced to the quadratic inequalities (6) to (10) in Table 2. For a particular frequency $_i$, there is a constant $_k=\ _k(\ _i)$, and for a fixed plant p in the $_i$-template and a fixed controller phase in [-360º,0º], the unknown parameter of the inequalities in Table 2 is the controller magnitude g. Then, solving equalities such as $ag^2+bg+c=0$ the set of $_i$-bounds for $\{\ _{k=1,..,5}\}$ is computed, leaving apart transitory the uncertainty $\{P\}$.

Table 2: Bound quadratic inequalities

k	Bound Quadratic Inequality	Eq.
1	$p^2\ \ 1\ \ \frac{1}{\ _1^2}\ \ g^2+\ 2\ p\ \cos(\ +\ \ )\ g+\ 1\ \ 0$	(6)
2	$p^2\ \ g^2+\ 2\ p\ \cos(\ +\ \ )\ g+\ 1\ \ \frac{1}{\ _2^2}\ \ 0$	(7)
3	$p^2\ \ g^2+\ 2\ p\ \cos(\ +\ \ )\ g+\ 1\ \ \frac{p^2}{\ _3^2}\ \ 0$	(8)
4	$p^2\ \ \frac{1}{\ _4^2}\ \ g^2+\ 2\ p\ \cos(\ +\ \ )\ g+\ 1\ \ 0$	(9)
5	$p_e^2p_d^2\ \ 1\ \ \frac{1}{\ _5^2}\ g^2+2p_ep_d\ \ p_e\cos(\ +\ _d)\ \ \frac{p_d}{\ _5^2}\cos(\ +\ _e)\ g+\ \ p_e^2\ \ \frac{p_d^2}{\ _5^2}\ \ 0$	(10)

Gil-Martínez and García-Sanz (2001) introduced the two possible solutions (G-bound formulation) to the quadratic inequalities in Table 2 for each feedback problem in Table 1. See Table 3. Choosing real and positive solutions g_{12} in Table 3 as effective controller restrictions, the same authors mentioned four ways to meet the bounds (i.e. the region on the NC to locate g), called typologies (see Figure 2). A bound plotted with a *solid line* implies that $G(j\ )$ (or $L_0(j\ )$) must lie above or on it to meet the particular specification, while a bound plotted with a *dashed line* means that $G(j\ )$ (or $L_0(j\ )$) must lie below or on it.

Table 3: G-bound formulation

k	$g_{1,2}$ bound formulas	Eq.
1	$g_{1,2}=\frac{1}{p\ \ 1\ \ \frac{1}{\ _1^2}}\ \ \cos(\ +\ )\mp\sqrt{\cos^2(+\ \ )\ \ 1\ \ \frac{1}{\ _1^2}}$	(11)
2	$g_{1,2}=\frac{1}{p}\ \ \cos(\ +\ )\mp\sqrt{\cos^2(+\ \ )\ \ 1\ \ \frac{1}{\ _2^2}}$	(12)
3	$g_{1,2}=\frac{1}{p}\ \ \cos(\ +\ )\mp\sqrt{\cos^2(+\ \ )\ \ 1\ \ \frac{p^2}{\ _3^2}}$	(13)
4	$g_{1,2}=\frac{1}{p\ \ 1\ \ \frac{1}{p^2\ \ _4^2}}\ \ \cos(\ +\ )\mp\sqrt{\cos^2(+\ \ )\ \ 1\ \ \frac{1}{p^2\ \ _4^2}}$	(14)
5	$g_{1,2}=\frac{1}{p_ep_d\ \ 1\ \ \frac{1}{\ _5^2}}\ \ p_e\cos(\ +\ _d)\ \ \frac{p_d}{\ _5^2}\cos(\ +\ _e)$ $\mp\sqrt{p_e\cos(\ +\ _d)\ \ \frac{p_d}{\ _5^2}\cos(\ +\ _e)\ \ 1\ \ \frac{1}{\ _5^2}\ \ p_e^2\ \ \frac{p_d^2}{\ _5^2}}$	(15)

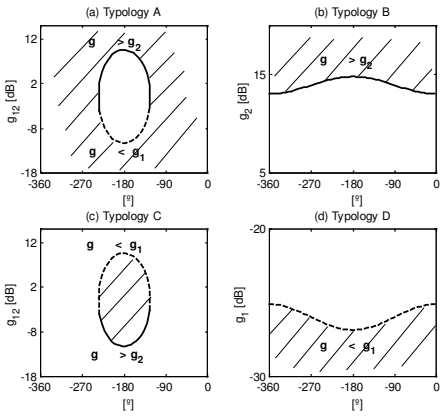

Figure 2: Bound typologies.

Computing the set of $\{g_{12}\}$ for all the $_i$-template $\{p \quad\}$, the less favourable (agreeing the typology), real and positive g_1 and/or g_2 can be calculated. Finally, Table 4 summarises the *G*-bound formulation and typology for the robust specifications in Table 1; (Gil-Martínez and García-Sanz, 2001). The setting of the bounds on $L_0=P_0\,G$ for a nominal plant $P_0\,\{P\}$, rather than on *G*, is more natural for the loop shaping in QFT. Then, being $L_0(j\ _i)=l_0\ _0$, the L_0-bounds are simply computed by translating the *G*-bounds vertically as $|P_0(j\ _i)|=p_0$ and horizontally as $P_0(j\ _i)=\ _0$. The results given for the *G*-bounds throughout the paper can also be applied to the L_0-bounds.

Table 4: Bound meeting for general feedback problems

k	$_k(\ _i)$	Typ	*G*-bound meeting; g_1 and g_2 in (11) to (15)
1	$0<\ _1<1$	D	$g \quad min\{g_1\} \quad = [\ 360º, 0º]$
	$_1>1$	A	$g \quad max\{g_2\} \quad g \quad min\{g_1\} \quad _{12}=[\ 180º\ \{\ \}\mp\ ]$
2	$0<\ _2<1$	B	$g \quad max\{g_2\} \quad = [\ 360º, 0º]$
	$_2>1$	A	$g \quad max\{g_2\} \quad g \quad min\{g_1\} \quad _{12}=[\ 180º\ \{\ \}\mp\ ]$
3	$p> \ _3$	B	$g \quad max\{g_2\} \quad = [\ 360º, 0º]$
	$\{p\}<\ _3$	A	$g \quad max\{g_2\} \quad g \quad min\{g_1\} \quad _{12}=[\ 180º\ \{\ \}\mp\ ]$
4	$p<1/\ _4$	D	$g \quad min\{g_1\} \quad = [\ 360º, 0º]$
	$\{p\}>1/\ _4$	A	$g \quad max\{g_2\} \quad g \quad g_1 \quad _{12}=[\ 180º\ \{\ \}\mp\ ]$
5	$\frac{p_{\max}}{p_{\min}}>\ _5$	B	$g \quad max\{g_2\} \quad = [\ 360º, 0º]$
	$\frac{p_{\max}}{p_{\min}}<\ _5$	A	$g \quad max\{g_2\} \quad g \quad min\{g_1\} \quad _{12}=[\ 180º\ \{\ \}\mp\ ]$

On the basis of this background, the following sections include a wide insight in role of $_k$ in the bound compatibility at each frequency and in the skills necessary to loopshape *G* along the frequency bounds.

3. SPECIFICATION VALUES AND BOUNDS

Checking the *G*-bound formulas in (11) to (15), note that for a specific compensator phase , the magnitudes g_1, g_2 are conditioned by the specification value $_k$, and the plant uncertainty: in magnitude $\{p\}$ and phase $\{\ \}$. The uncertainty is inherent to the nature of the process, and then, it should be considered as it is. On the other hand, the specifications $_{k=1,...,5}$ are design parameters to obtain the best robust performance simultaneously achievable. The contribution of the $_k$-value to the *G*-bounds is two-folded.

First, $_k$ conditions the bound typology (see Table 4). Obviously, the bound type determines the compatibility of the set of $_i$-bounds (k=1,...,5 bounds).

The other $_k$ contribution focuses on the bound aggressiveness in terms of height (g_{12} magnitudes) and width ($_{12}$ phase range). In the typology A (Figure 2*a*), $g \quad min\{g_1\}$ and $g \quad max\{g_2\}$, $_{12}=[\ 180º\ \{\ \}\mp\ ]$, the bound severity increases when: (*a*) its existing phase range $_{12}$ ($\{\ \}$ or) increases, (*b*) the magnitudes g_1 decrease and the magnitudes g_2 increase at each $_{12}$ for $\{p \quad\}$. In the typology B (Figure 2b), $g \quad max\{g_2\}$, $=[-360º, 0º]$, the bound is more severe when the magnitudes g_2 increase at each for $\{p \quad\}$. In the typology D (Figure 2*d*), $g \quad min\{g_1\}$, $=[-360º, 0º]$, the bound hardens when the values g_1 decrease at each for $\{p \quad\}$.

The figures $_{k=1,...,5}$ are following analyzed, proving their role on the typology of the bound (bound shape) and on its aggressiveness (bound height and width).

3.1 Specification values on the complementary sensitivity function, $|L/1+L|\ _1$

As shown in Table 4, depending on the specification value, constant at $_i$ as $_1=\ _1(\ _i)$, there are two different bound typologies in (11): D for $0<\ _1<1$ (Case I), and A for $_1>1$ (Case II). To ensure reasonable phase and gain margins (*PM, GM*) in terms of robust stability ($|L/1+L|<\ _1$), a minimum value of $_1>1.3$ is advisable. It ensures *PM* 45º and *GM* 5dB (Chait and Yaniv, 1993). Therefore, a $_1=\ _1(\ _i)$ 1.3>1 at the set $\{\ _i\}$ ensures that the stability bounds are always of type A (Case I). In general, the specifications for robust sensor noise attenuation ($|Y/N|<\ _1$) are watched at high frequencies and usually satisfy $0<\ _1<1$, since the noise effect must be attenuated at the output. Then, they provide type D bounds. As regards the control effort allowed for the input disturbance rejection ($|U/D_1|<\ _1$), $_1$ can take any value higher than 0, and therefore produce bounds of type B or A.

Case I: $0<\ _1<1$, *G-Bounds of type D*. When the specification value $_1=\ _1(\ _i)$ decreases in (11), the magnitude g_1 drops in [-360º, 0º], and then, the bound condition is more severe. See Figure 3*a*.

Case II: $_1>1$, *G-Bounds of type A*. Checking (11), a decrease in $_1$ implies, an increase in (bounds at a wider phase band), g_2 rises and g_1 is reduced. Then, the bounds are more severe when the specification figures of merit decrease, as shown in Figure 3*b*.

In summary, the smaller the specification value is, the more aggressive the corresponding bound in both cases is, namely bounds of type A or D. A decrease in $_1$ also implies a transition to a harder typology (A to D) at $_1=1$. A smaller $_1$ tolerance involves harder restrictions on the stability and the performance: larger phase and gain margins, a smaller control effort for the

disturbance rejection or a higher noise attenuation level required in the output. Its QFT representation through the bounds in Figure 3 shows it.

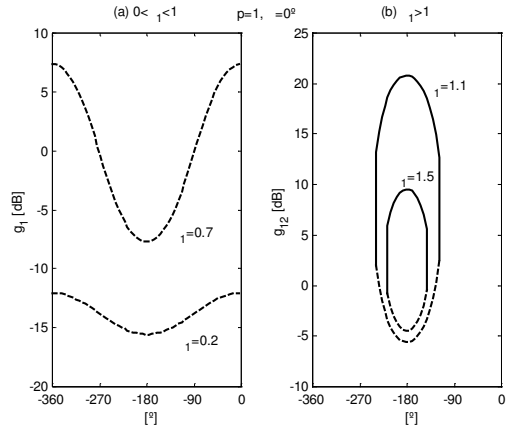

Figure 3: α_1 influence on its representative bound.

3.2 Specification on the sensitivity function, $|1/1+L|$ α_2

Depending on the specification value at ω_i, $\alpha_2=\alpha_2(\omega_i)$, two different bound typologies in (12) will appear: B for $0<\alpha_2<1$ (Case I), and A for $\alpha_2>1$ (Case II); see Table 4. In line with (2), $|Y/D_2|<\alpha_2$ expresses the robust performance in the rejection of output disturbances. Then, α_2 usually takes attenuation values $0<\alpha_2<1$, giving type B bounds. The sensitivity values $|S|<<1$ at some low frequencies need, at least in practice, that $|S|>1$ at some moderate high frequencies (Skogestad and Postlethwaite, 1996). This S peak must be constrained by $\alpha_2>1$ but close to 1 to ensure minimum gain and phase margins.

Case I: $0<\alpha_2<1$, *G-Bounds of type B*. When the specification α_2 decreases, the magnitude g_2 increases in [-360º, 0º], and thus the bound condition is more severe. Figure 4*a* shows these results. Smaller figures of merit α_2 imply a more aggressive specification in order to obtain a larger D_2 attenuation level at Y.

Case II: $\alpha_2>1$, *G-Bounds of type A*. The role of $\alpha_2>1$ is the same than the one described in Section 3.1 for the Case I, $\alpha_1>1$. Larger stability margins (α_2 decrement) imply stronger type A bounds. See Figure 4*b*.

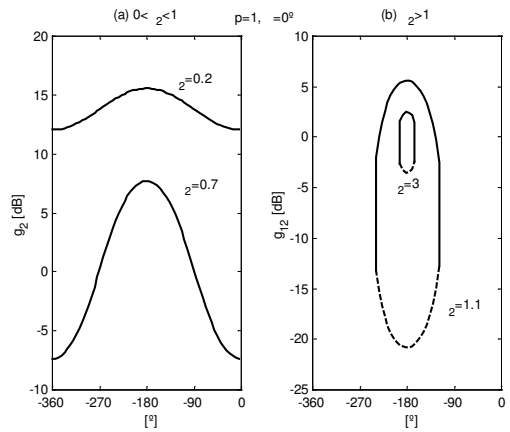

Figure 4: α_2 influence on its representative bound.

3.3 Specification values on the sensitivity plus the plant function, $|P/1+L|$ α_3

The equation (13) in Table 3 provides two different bound typologies which depend on the relation between the constant specification $\alpha_3=\alpha_3(\omega_i)$, and the plant magnitude $\{p(\omega_i)\}$. A typology B appears if $p\ \{p\}/p>\alpha_3$ at ω_i (Case I), and a typology A exists if $p\ \{p\}/p<\alpha_3$ (Case II). Since $|Y/D_1|=|P/1+L|$ α_3 expresses the attenuation in the plant output of disturbances at its input, the α_3-tolerance usually takes values of $0<\alpha_3<1$. It may produce type B or type A bounds. Since the smaller α_3 and bigger $\{p\}$ produce at low frequencies, type B bounds are more usual at these frequencies. Smaller α_3 imply stronger B or A bounds which involve a larger disturbance attenuation. The bound appearance for the α_3 variation is the same than the one for the α_2 specification in Figure 4.

3.4 Control effort restrictions, $|G/1+L|$ α_4

As shown in Table 4, depending on the relation between the specification, $\alpha_4=\alpha_4(\omega_i)$, and the plant magnitude $\{p(\omega_i)\}$, the equation (14) offers two different typologies: D when $p\ \{p\}/p<1/\alpha_4$ (Case I), and A if $p\ \{p\}/p>1/\alpha_4$ (Case II). Note that α_4 can take any value >0, generating type D or A bounds. Smaller α_4-values imply tougher D or A bounds, i.e. larger restriction on the control effort, usually required with increasing frequencies. The bound variation looks like the α_1-specification influence shown in Figure 3.

3.5 Signal Tracking, α_{5inf} $|L/1+L|$ α_{5sup}

The particular performances of the signal tracking and the disturbance attenuation require two degrees of freedom (Horowitz, 1963). The goal of the controller in the tracking is to reduce the closed loop T-variation, being $|T|=|PG/(1+P\ G)|$, due to the uncertainty in P. At the same time, G should achieve the remaining performance and stability specifications. At a second design step, a pre-filter F positions $|T_5|=|F\ T|$ according to the upper α_{5sup} and lower α_{5inf} models in (5). In the task of the G-controller design, (5) can be rewritten as:

$$\frac{|T_d(j\ \omega_i)|}{|T_e(j\ \omega_i)|}\quad\frac{\alpha_{5sup}(\omega_i)}{\alpha_{5inf}(\omega_i)}=\alpha_5(\omega_i)=\alpha_5 \qquad (16)$$

assuming that $|T_d(j\ \omega_i)|>|T_e(j\ \omega_i)|$. Since $\alpha_{5sup}>\alpha_{5inf}$, then $\alpha_5(\omega_i)>1$. A α_5-decrement is aimed at obtaining a larger reduction in the T-variation for the same uncertainty in P. Then, the specification is more severe, and its QFT bound behaves correspondingly.

According to the bound formula in (15), the relation between $\alpha_5(\omega_i)$ and the plant maximum magnitude uncertainty, $p_{max}/p_{min}(\omega_i)$, determines two different typologies of bounds (see Table 4). The type B for $p_{max}/p_{min}>\alpha_5$ (Case I), and the type A for $p_{max}/p_{min}<\alpha_5$ (Case II). Let's assume p_{max} and p_{min}, the plants with maximum and minimum modulus in $\{p(\omega_i)\}$.

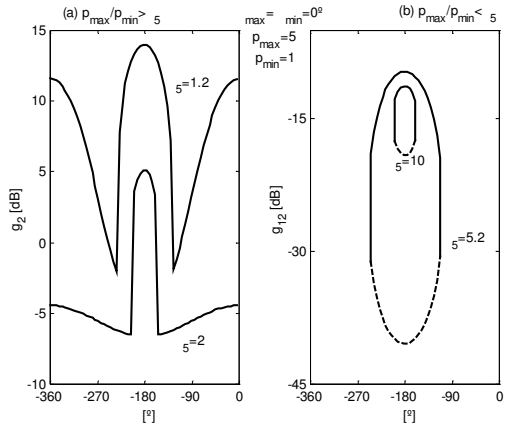

Figure 5: $_5$ influence on its representative bound.

Case I: $p_{max}/p_{min}>$ $_5$, *G-Bounds of type B*. When the specification $_5=\ _5(\ _i)$ decreases (major uncertainty reduction required), the magnitude g_2 in (15) decreases at $\in$ [-360º, 0º], so the bound condition hardens. See Figure 7*a*.

Case II $p_{max}/p_{min}<$ $_5$, *G-Bounds of type A*. When $_5$ decreases: increases, g_2 rises and g_1 decreases; then, the bound fulfillment toughens. Check Figure 7*b*. The type A bound relaxation at high frequencies allows closed loop uncertainties larger than those of the open loop, with a negligible feedback effect

4. CONTROLLER DESIGN STRATEGIES

The bounds are the crux of the solution to the robust feedback problem in QFT from two different points of view. Firstly, it is necessary to have bound compatibility. This means that at each frequency $_i$, there must be a non-null bound intersection amongst the bounds expressing the different k robust specifications. Secondly, it must be possible to loopshape a nominal open loop transmission, $L_0(\mathrm{j}\ _i)=G(\mathrm{j}\ _i)P_0(\mathrm{j}\ _i)$, meeting the non-null bound intersection along the set of frequencies $\{\ _i\}$.

Gil-Martínez and García-Sanz (2001) partially dealt with the first question. They claimed that to meet multi-objective bounds at a particular frequency simultaneously, type A bounds of any magnitude $g_{1,2}$ can coexist with type B or D bounds. If type B and D bounds coexist, the D-bound g_1 magnitudes should be higher than the B-bound g_2 magnitudes for some , relaxing $_k$ when necessary. This is a consequence of the trade-offs in the feedback control, which harden in the case of the uncertain systems. The present paper includes some more insights in the matter.

After establishing that the bound typology has a strong influence on the existence of a global solution, note that: (*a*) the bound typology of the feedback problems k=1,2 depends exclusively on the desired behavior $_k$; (*b*) in contrast, for k=3,4, the bound typology is not only determined by the desired specification, but also by the uncertainty inherent to the system; (*c*) the feedback benefits at low and medium frequencies expressed through specifications k=2,3,5 usually lead to type B bounds; (*d*) as a particular case of the feedback problem k=1, robust stability constraints appear in any control problem, but fortunately, they provide typology A bounds that are compatible with other typologies.

The bound magnitude g_{12} at each and the bound phase range $_{12}$ are other relevant factors in the bound solution. Section 3 showed the influence of $_k$ in this case. As expected, stronger $_k$ figures of merit lead to more severe bounds expressing them in QFT (see Figures 3, 4, 5).

Assuming a bound compatibility at each frequency, the bound severity increase due to the harder $_k$ tolerances implies a more challenging loopshaping of the controller. The following example proves it. Let's suppose an uncertain plant family $\{P(\mathrm{s})=k/(\mathrm{s}/a+1),\ k\ \in [1, 5],\ a\ \in [1, 5]\}$. Two frequencies are evaluated, a low frequency $_{lf}$=1rad/s and a high frequency $_{hf}$=100rad/s. Figure 6 shows the plant templates, with a nominal plant $P_0(\mathrm{j}\ )$ for k=1, a=1. The feedback requirements are: (*a*) a robust stability with $_1(\ _i)=\ _1$=1.3 for all frequencies, in particular, $_i$={1,100} rad/s. This ensures for the whole set of plants that *PM* 45º and *GM* 5dB. (*b*) A robust disturbance rejection at low-medium frequencies, gained with a suitable $_2(\ _{lf})$ tolerance at $_{lf}$=1 rad/s. This imply a comparison between $_2^{\mathrm{I}}(\ _{lf})$=0.3 and $_2^{\mathrm{II}}(\ _{lf})$=0.1. The first one assures a disturbance attenuation level of $|Y/D_2|$ -10.5dB, and the second one of -20dB.

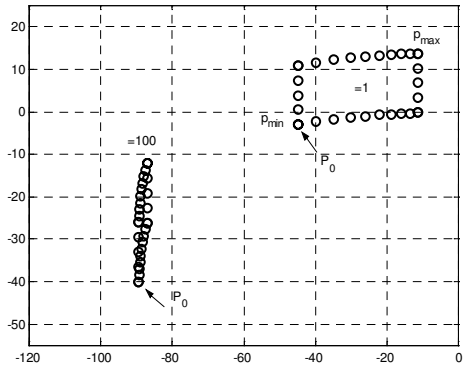

Figure 6: Plant templates.

According to Table 4, since $_2^{\mathrm{II}}(\ _{lf})$=0.1 and $_2^{\mathrm{I}}(\ _{lf})$=0.3 are lower than 1, they imply typology B bounds. In keeping with Figure 4*a* and Section 3.2, a decrease in $_2$ implies an increase in the bound height (severity), particularly around 0º and –360º: $_2^{\mathrm{II}}$ implies a more stringent requirement than $_2^{\mathrm{I}}$ in the disturbance attenuation. As a consequence of it, a larger controller static gain is required (see Figure 7). The loopshaping, $L_{0I}=G_1P_0$, provides a controller for the weaker specification ($_2^{\mathrm{I}}(\ _{lf})$=0.3):

$$G_1(s)=\frac{4}{s/62+1} \qquad (17)$$

The strongest performance requirement ($_2^I$($_{lf}$)=0.1) results in $L_{02}=G_2P_0$ or $L_{03}=G_3P_0$ with controllers:

$$G_2(s) = \frac{15}{s/247+1} \tag{18}$$

$$G_3(s) = \frac{15(s/96+1)}{(s/36+1)(s^2/465^2 + 2\ \ 0.7s/465+1)} \tag{19}$$

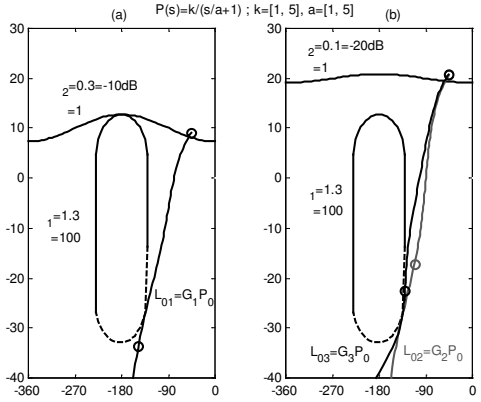

Figure 7: Bounds and loopshaping for robust stability and output disturbance rejection.

An improvement in the performance of the disturbance rejection requires a larger controller static gain (11dB increment between G_2 or G_3 and G_1 gains), claimed by a higher $_{lf}$-bound. The drawbacks turn up at high frequencies (see Figure 7), where $|L_0($ $_{hf}$=100)| raises from –33.6dB for L_{01} to –22.3dB for L_{03} or to –17dB for L_{02}. Figure 8 shows some comparative performances between the G_1 and G_2 controllers in the time domain for the nominal plant P_0=1/s+1 (plant with the slowest dynamic response). A step disturbance, $D_2(s)$=1/s, appears at t=1sec., being the reference $R(s)$=0. The sensor noise N(s) is simulated by means of a noise disturbance signal with a correlation time 100 times smaller than the fastest dynamics of the system. Figure 8*a* shows the larger disturbance level attenuation gained with G_2 against G_1, although it also implies a slightly larger noise amplification at the system output. Figure 8*b* clearly proves the main feedback drawback: a high frequency $|L_0|$ mainly leads to large $|U/N|$ values that can saturate the actuators (Horowitz, 1973). Better feedback benefits at low frequencies and a relative small high frequency $|L_0|$ (to avoid $|U/N|$ peaks) are simultaneously possible by increasing the complexity of the controller. Compare G_3 with G_2 in Figure 7. However, a higher controller order also implies larger delays in the right control application in practice. The G_2-controller simplicity implies overdesign in stability margins (23º in *PM* and 17dB in *PM*).

5. CONCLUSIONS

The more severe performance and stability specifications in SISO uncertain plants were required, the more stringent feedback trade-offs were proven. Firstly, the bound aggressiveness was studied. Secondly, the controller synthesis difficulties were detailed in a practical example.

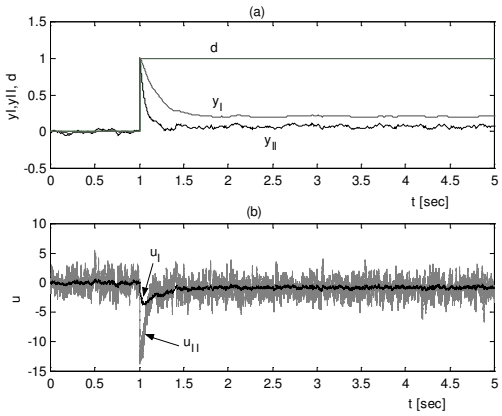

Figure 8: Time domain performance for $P_0(s)$=1/*s*+1.

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