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Multiproduct CVP analysis based on contribution rules

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Abstract

The basic cost-volume-profit (CVP) model – linear, non-stochastic and restricted to one product – has been the subject of research work aimed at relaxing these limiting assumptions. Regarding its extension to a multiproduct situation, the two alternatives are to use a standard mix, or to apply linear programming. This paper develops an alternative model for multiproduct CVP. It uses data provided by ABC systems designed to keep track of variable and fixed costs, and requires the model user to formulate a contribution rule that will allow to compute, for each product, the output required to achieve a given (target) profit. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Cost-volume-profit analysis (CVPA) studies the relationships between costs, revenue (as a function of output volume and selling price), and profit. The analysis usually focuses on the computation of two specific output values: the one required to achieve a given target profit, and the one for which the business breaks even. In both cases, CVPA works as a management-goal setter. Ijiry [1] discussed in a classical piece of work the value of CVPA in this regard. After these two values have been computed, it is straightforward to obtain the margin of safety. The relative margin of safety, which is the inverse of the output elasticity of profit (which, in turn, corresponds to the operating leverage) would be a measure of the risk associated with those two output values.

The basic CVP model is based on three assumptions: (i) one single product is manufactured and sold, (ii) both the total cost and total revenue functions are linear, and (iii) the fixed costs, the average variable cost per unit, and the selling price are known with certainty. For nearly forty years this basic model – linear, non-stochastic and restricted to one product – has been the subject of research work aimed at addressing the relaxation of one or another of these three hypotheses.

Concerning the second assumption, the general approach in management accounting is to hold on to the linearity assumption. The parameters of the curves (for cost and revenue) would be very difficult to estimate in practice. In addition, there is a range of operations for which the economists' cost curve and the accountants' are basically coincident (where short-term average costs are minimum), and that is where companies will generally try to

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position themselves, thus constituting the relevant range for short term analysis. However, it should be said that linear and curvilinear (cost and revenue) functions are not the only two choices. Ijiry [1] developed an 'extension to a piecewise linear model' that factors in different cost behaviors (variable, fixed, semi-variable and semi-fixed), and a revenue function where price changes discretely. An approach enriched by Tsai and Lin [2] through extension to multiproduct CVPA.

The introduction of uncertainty in the parameters (basically, the output) in the model has been, and continues to be, a frequent topic in management accounting literature (see Scapens [3], for a review of this line of research). However, the difficulties of integrating uncertainty into the model, especially in a multiproduct setting [3], have led to an emphasis on sensitivity analysis.[1]

But it is the relaxation of the first hypothesis, arguably the one that poses the biggest difficulty, that this paper is concerned with.

The problem with extending the basic model to a multiproduct situation is that there is a huge (if not infinite) number of feasible mixes for which the organization would break even (or would reach a given target profit). There are two ways to determine a unique solution in this case:[2]

i. To use a standard product mix (usually, the planned or the expected mix).
ii. To factor in capacity constraints and find an optimal solution through application of operational research techniques (see Kaplan [5], for an in-depth discussion of this methodology).

Both approaches have limitations. Arguably, the standard mix should not be applied to a set of products unless their demands are clearly correlated. In other words, it should be limited to similar products to be distributed in similar markets. Thus, it would not seem sensible for a global conglomerate to apply the standard mix approach at enterprise level.

As regards the second approach, it raises two problems. To begin with, it is only applicable if there are measurable non-financial production constraints (sometimes referred to as resources constraints). That is, only if all of the products in the range have some limited input in common.[3] The second limitation is that linear programming-based CVPA can only yield, by definition, solutions that are optimal one way or another, thus implying that production is supply-driven. An assumption that is both, unrealistic and inconsistent with CVPA objectives which are, rather than to compute optimal output values, to translate into a production-and-sales goal the target profit, to establish a "floor" for performance (the breakeven point), and to give an estimate of the risk associated with the former as measured by the difference between it and the later.

In the coming sections, a third approach, based on the application of contribution rules formulated by the model user, is discussed.

2. Basic contribution rules-based CVPA

Assume a business manufactures in l different industrial plants, $P = \{p_i : i = 1, 2, \dots, l\}$, m different products, $X = \{x_j : j = 1, 2, \dots, m\}$, which are then sold through n different marketing channels (sales outlets or combinations of them), $C = \{c_k : k = 1, 2, \dots, n\}$. Assume as well that each product is only manufactured at one facility,[4] so that the set P is in effect a partition of the set X (that is, P is a collection of non-empty disjoint subsets of X, whose union is X). On the other hand, a product may be sold through any of the n defined marketing channels.

[1] When conducted under different 'scenarios' (combinations of output volume, selling price, and costs), CVPA is called (profit) sensitivity or 'what if' analysis [4]; and when safety margin (or its inverse, the operating leverage) is considered, it becomes a powerful tool for risk analysis.

[2] Tsai and Lin [2] solve their model for a given target profit (which may be zero). Their solution has, though, two shortcomings: (1) uniqueness of the solution is not guaranteed, and (2) it does not contemplate the existence of unallocated fixed costs.

[3] The input available quantity and the rate of consumption for each unit of the different products have to be known as well. This makes this technique difficult, if not impossible, to apply outside manufacturing.

[4] Though very restrictive, this assumption is not essential to the model, it is made for simplicity.

If the business in question has an ABC system in place, designed to keep track of variable and fixed costs,[5] the following data should be available:[6]

- Costs of business-sustaining activities, which may be characterized as fixed costs: $F(B)$.
- For each manufacturing center, (fixed) costs of facility-sustaining activities: $F(p_i)$.
- Costs related to each marketing channel, which may be assumed to be fixed[7] (in the short term): $F(c_k)$.
- For each product, product fixed costs, $F(x_j)$, defined as the sum of: (i) costs of product-sustaining activities, plus (ii) costs of non-flexible resources committed to batch- and unit-related activities exclusively associated with the manufacturing of product x_j.
- For every product, variable costs of batch- and unit-related activities, plus direct costs (materials and, only if there is full hire-and-fire flexibility, direct labor), per unit of output: $v(x_j)$.

In addition, there may be non-flexible resources, associated with batch- and unit-related activities, that can be applied to the manufacturing of different products. This possibility will not be considered for the time being. Likewise, capacity constraints will not be taken into account. Nor, for that matter, the possible existence of products with non-positive contribution margins, that is, products for which $v(x_j) \geqslant p(x_j)$, where $p(x_j)$ stands for the unit selling price.

Whichever approach is applied, the analysis should provide the quantity of each product, $Q(x_j)$, to be manufactured and sold, so that all costs are recovered and a certain profit, π (zero for the breakeven point), is derived from the business activity. That is

$$\sum_{j=1}^{m} Q(x_j)(p(x_j) - v(x_j)) = F(B) + \sum_{i=1}^{l} F(p_i) + \sum_{k=1}^{n} F(c_k) + \sum_{j=1}^{m} F(x_j) + \pi. \quad (1)$$

Eq. (1) has, at best (if $Q(x_j) \in \mathrm{II}, j = 1, 2, \ldots, m$), a huge number of valid solutions. The next step, according to the approach herein discussed, would be for the model user to formulate a *contribution rule* (Fig. 1).

i. What products will contribute to cover which costs.

ii. When a specific cost item is to be covered by more than one product (as will be the case with those associated with business, channel, and facility-sustaining activities), what basis is to be used to compute each product's contribution.

iii. In what proportion will contribute each product to the target profit, if any.

Fig. 1. Contribution rule (basic clauses).

Thus, to begin with, a contribution rule might have the following content:

1. Each product, x_j, must cover those costs, both fixed and variable, that are specific to its production, $F(x_j) + Q(x_j)v(x_j)$.
2. Facility-sustaining costs, $F(p_i)$, shall be covered by the products manufactured at each facility, $x_j \in p_i$.
3. Channel-sustaining costs, $F(c_k)$, should be covered by the products to be distributed through each channel.
4. All the products should contribute to business-sustaining costs, $F(B)$, and profit, π.

In addition, it is necessary to establish, for items 2–4 above, the bases for contribution apportionment to the different products, so that the following values may be computed:

- Proportion, $\phi(x_j)$, of $F(p_i)$ to be covered by $x_j \in p_i$

  $\phi(x_j), \ j = 1, 2, \ldots, m,$

  $$\text{such that: } \sum_{x_j \in p_i} \phi(x_j) = 1, \ i = 1, 2, \ldots, l. \quad (2)$$

[5] Regarding ABC refinement to allow for contribution margin analysis, see [6–8].

[6] The hierarchy of activities applied here includes the four initially formulated by Cooper [9], plus business and channel-sustaining activities. A complete hierarchy may be obtained from [10].

[7] Otherwise, the variable costs per unit sold may be added to $v(x_j)$, given the assumption, usual in CVPA, of zero increase in product-stocks.

- Proportion, $\chi_k(x_j)$, of $F(c_k)$ to be covered by x_j

$\chi_k(x_j),\ j = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, n$,

$$\text{such that: } \sum_{j=1}^{m} \chi_k(x_j) = 1,\ k = 1, 2, \ldots, n. \tag{3}$$

- Proportion, $\beta(x_j)$, of $F(B) + \pi$, to be covered by x_j

$\beta(x_j),\ j = 1, 2, \ldots, m$,

$$\text{such that: } \sum_{j=1}^{m} \beta(x_j) = 1. \tag{4}$$

Facility-sustaining costs are independent of the actual output, they are rather a function of the (manufacturing) installed capacity. Hence, three possible bases to compute the values in Eq. (2) could be

i. Product's fixed costs

$$\phi(x_j) = \frac{F(x_j)}{\sum_{x_h \in p_i | x_j \in p_i} F(x_h)},\ j = 1, 2, \ldots, m. \tag{5}$$

ii. Total product's costs at practical capacity[8]

$$\phi(x_j) = \frac{F(x_j) + Y(x_j)v(x_j)}{\sum_{x_h \in p_i | x_j \in p_i} F(x_h) + Y(x_h)v(x_h)},$$

$$j = 1, 2, \ldots, m, \tag{6}$$

where $Y(x_j), j = 1, 2, \ldots, m$, stands for the practical capacity for product x_j.

iii. Invested capital specifically associated with each product's manufacture, $K(x_j), j = 1, 2, \ldots, m$,

$$\phi(x_j) = \frac{K(x_j)}{\sum_{x_h \in p_i | x_j \in p_i} K(x_h)},\ j = 1, 2, \ldots, m. \tag{7}$$

If the second alternative is chosen, it would be necessary to know the values of $Y(x_j)$, $j = 1, 2, \ldots, m$. If it is the third one that is selected, the information required would include the invested capital for each product, defined as the net value of the corresponding fixed assets, plus an estimate of the associated working capital. A problem arises here, because of the working capital being a function, among other things, of the output.[9] It is the later which is precisely to be computed, hence, only by trial and error will it be possible to proceed, unless the said estimate is made for the production at practical capacity, or a percentage thereof.

Regarding channel-sustaining costs, one possible way to operate is as follows:

1. Estimate for each product the proportion of sales that take place through each channel.

$$\begin{array}{lccccccc}
 & & & & \text{Products} & & & \\
 & & 1 & \cdots & j & \cdots & m \\
 & 1 & \alpha_{1,1} & \cdots & \alpha_{1,j} & \cdots & \alpha_{1,m} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\text{Channels} & k & \alpha_{k,1} & \cdots & \alpha_{k,j} & \cdots & \alpha_{k,m} \\
 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 & n & \alpha_{n,1} & \cdots & \alpha_{n,j} & \cdots & \alpha_{n,m}
\end{array}$$

$$\sum_{k=1}^{n} \alpha_{k,j} = 1,\ j = 1, 2, \ldots, m.$$

It should be easy to obtain these values, based on previous years sales.

2. If it is assumed that sales value is an adequate basis to distribute the costs of a marketing channel among the products sold through it, then products should contribute to channel-sustaining costs in the following proportions (values in Eq. (3)):

$$\chi_k(x_j) = \frac{\alpha_{k,j} Q(x_j) p(x_j)}{\sum_{h=1}^{m} \alpha_{k,h} Q(x_h) p(x_h)},$$

$$j = 1, 2, \ldots, m \text{ and } k = 1, 2, \ldots, n. \tag{8}$$

Note that sales quantities are determined by the costs to be covered, allotted channel-sustaining costs included. Given that they are unknown at this stage, the same problem brought up in connection with the working capital arises here. Again, only by

[8] Practical capacity may be defined as the available capacity given the actual operating conditions [11].

[9] The other main factors affecting the amount of required working capital being turnover rates of inventories, and terms of purchase and of credit sales.

trial and error will the computation be possible. This difficulty may be overcome if the desired values are calculated assuming sales at (manufacturing) practical capacity

$$\chi_k(x_j) = \frac{\alpha_{k,j} Y(x_j) p(x_j)}{\sum_{h=1}^{m} \alpha_{k,h} Y(x_h) p(x_h)}, \qquad j = 1, 2, \ldots, m \text{ and } k = 1, 2, \ldots, n, \tag{9}$$

which would imply a different rationale, on the other hand.

As regards business-sustaining costs, it seems reasonable to apply the same product-allotment basis for these costs and for the target profit. Two possible (and sensible) alternatives to compute each product's contribution are:

i. Proportional to its contribution capability, defined as the margin at capacity, after deducting all other costs to be covered by it (share of facility-sustaining and channel-sustaining costs included)

$$\beta(x_j) = \frac{Y(x_j)(p(x_j) - v(x_j)) - F(x_j) - \phi(x_j)F(p_i | x_j \in p_i) - \sum_{k=1}^{n} \chi_k(x_j) F(c_k)}{\sum_{h=1}^{m} Y(x_h)(p(x_h) - v(x_h)) - F(x_h) - \phi(x_h)F(p_i | x_h \in p_i) - \sum_{k=1}^{n} \chi_h(x_h) F(c_k)}, \qquad j = 1, 2, \ldots, m. \tag{10}$$

ii. In proportion to the invested capital directly or indirectly associated with its production and distribution, $K'(x_j)$

$$\beta(x_j) = \frac{K'(x_j)}{\sum_{h=1}^{m} K'(x_h)},$$

$$K'(x_j) = K(x_j) + \phi(x_j) K(p_i | x_j \in p_i) + \sum_{h=1}^{m} \chi_k(x_j) K(c_k), \quad j = 1, 2, \ldots, m, \tag{11}$$

where $K(p_i | x_j \in p_i)$ is the invested capital for plant i, where product j is manufactured, excluded capital directly associated with one product or another of those manufactured at the facility, and $K(c_k)$ stands for the capital associated with channel k.

The user of the model will have to formulate the contribution rule as a combination of the alternatives discussed above, or variations of them (for instance, by weighting product's practical capacity, where applicable, with a given value, $0 < \lambda_j \leqslant 1$, in order to factor in estimated sales constraints).

Different rules will obviously lead to different results, and it is the model user who shall formulate the rule, consistently with the characteristics of the business and its operating environment. As a rule of thumb, it seems reasonable to postulate that contribution rules should somehow reflect the criteria for judging product's performance.

In addition to all this, there would still remain some additional questions to be determined when formulating the rule. As was duly pointed, there are three issues which have been disregarded in this section, namely: (i) capacity (and sales) constraints, (ii) products with non-positive margins, and (iii) existence, at factory level, of resources applicable to the production of different products. These three questions need to be addressed when formulating the contribution rule of choice. The first two are discussed in Section 4, while Section 5 deals with the third one. But before that, Section 3 below illustrates with a numeric example the basic model that has been discussed in the running section.

3. A numeric example

Table 1 shows hypothetical data for a business that manufactures four different products at two different locations. Products 1 and 2 are manufactured at facility #1, and the other two at facility #2. Please, note that the values (1) through (5) are shown for each product and, where applicable, for each of the two manufacturing plants. In practice, these would be the expected values for the period under analysis.

Table 2 details the computation of each product's contribution to facility-sustaining costs. Of the three alternatives discussed in Section 2, it is the second one (Eq. (6)) that has been used.

Table 1

Key	Description	Facility #1	Product 1	Product 2	Product 3	Product 4	Facility #2
1	Fixed costs	$540,000	$1,750,000	$1,200,000	$1,140,000	$250,000	$483,850
2	Invested capital	$1,650,000	$3,000,000	$2,500,000	$850,000	$945,000	$538,500
3	Capacity (units)		50,000	60,000	30,000	25,000	
4	Price per unit (p.u.)		$140	$135	$146	$150	
5	Variable costs p.u.		$25	$35	$28	$60	

Table 2

Key	Description	Product 1	Product 2	Product 3	Product 4
6	Total product's costs at capacity	$3,000,000	$3,300,000	$1,980,000	$1,750,000
7	Contribution to facility-sustaining costs	$257,143	$282,857	$256,843	$227,007

Table 3

Channels	Product 1	Product 2	Product 3	Product 4	Fixed costs	Inv. capital
Sales region A	22%	14%	44%	12%	$555,632	$1,010,240
Sales region B	44%	44%	44%	29%	$1,239,533	$1,609,783
Sales region C	34%	42%	12%	59%	$1,168,471	$1,217,157
Totals	100%	100%	100%	100%	$2,963,636	$3,837,180

Table 4

Sales at capacity	Product 1	Product 2	Product 3	Product 4	Totals
Sales region A	$1,540,000	$1,134,000	$1,927,200	$450,000	$5,051,200
Sales region B	$3,080,000	$3,564,000	$1,927,200	$1,087,500	$9,658,700
Sales region C	$2,380,000	$3,402,000	$525,600	$2,212,500	$8,520,100
Totals = (3) × (4)	$7,000,000	$8,100,000	$4,380,000	$3,750,000	$23,230,000

For each product: $(6) = (1) + (3) \times (5)$. Then, the values in (1) for each facility are distributed in proportion to (6) among the relevant products (fac #1's costs between products 1 and 2, and fac #2's costs between products 3 and 4).

Table 3 contains all the available data referred to the three existing channels for the marketing and distribution of the manufactured products.

Besides fixed costs and invested capital associated with each channel, Table 3 gives, for each product, the proportion of sales per channel (based on previous periods data).

If sales at capacity is computed for each product, $(3) \times (4)$, and the resulting values are multiplied by the percentages in Table 3, the values in Table 4 are obtained.

Eq. (9), discussed in the previous section, yields, when applied to the data shown in Table 4, the values disclosed in Table 5.

That is, the proportion of each channel's sustaining costs to be covered by each product. These results allow to compute those in row (8) in Table 6.

After contribution to channel-sustaining costs are computed, it is possible to calculate the available contribution capability of each product: $(10) = (3) \times ((4) - (5)) - (9)$, so that the contributions to business-sustaining costs plus profit may

Table 5

$\chi_k(x_j)$	Product 1	Product 2	Product 3	Product 4	Totals
Sales region A	30.49%	22.45%	38.15%	8.91%	100%
Sales region B	31.89%	36.90%	19.95%	11.26%	100%
Sales region C	27.93%	39.93%	6.17%	25.97%	100%

Table 6

Key	Description	Product 1	Product 2	Product 3	Product 4	Totals
8	Contribution to channel-sustaining costs	\$891,067	\$1,048,680	\$531,398	\$492,491	\$2,963,636
9	(1) + (7) + (8)	\$2,898,210	\$2,531,537	\$1,928,241	\$969,498	\$8,327,486
10	Available contribution capability	\$2,851,790	\$3,468,463	\$1,611,759	\$1,280,502	\$9,212,514
11	Contribution to business-sust. costs plus profit	\$1,672,008	\$2,033,564	\$944,976	\$750,760	\$5,401,309
12	(9) + (11)	\$4,570,218	\$4,565,101	\$2,873,218	\$1,720,258	\$13,728,795
13	(12) ÷ ((4) − (5))	39,741	45,651	24,349	19,114	
14	(13) ÷ (3)	79.48%	76.09%	81.16%	76.46%	

Table 7

Key	Description	Product 1	Product 2	Product 3	Product 4	Totals
15	Invested capital adjusted with facility inv. cap.	\$3,900,000	\$3,250,000	\$1,105,000	\$1,228,500	\$9,483,500
16	Contribution to facility-sustaining costs	\$294,545	\$245,455	\$229,121	\$254,729	
17	Contribution to channel-sustaining costs	\$891,067	\$1,048,680	\$531,398	\$492,491	
18	Channel invested capital adjustment	\$1,161,333	\$1,306,800	\$781,726	\$587,321	\$3,837,180
19	Inv. cap. adjusted with fac. and chan. inv. cap.	\$5,061,333	\$4,556,800	\$1,886,726	\$1,815,821	\$13,320,680
20	Contribution to business-sust. costs plus profit	\$2,052,284	\$1,847,705	\$765,035	\$736,285	
21	(1) + (16) + (17) + (20)	\$4,987,897	\$4,341,839	\$2,665,555	\$1,733,505	
22	(21) ÷ ((4) − (5))	43,373	43,418	22,589	19,261	
23	(22) ÷ (3)	86.75%	72.36%	75.30%	77.04%	

be computed applying Eq. (10) (Section 2). Assuming business-sustaining costs of \$3,270,000, and a target profit of \$2,131,309, the values in row (11) are the contributions required to cover these two amounts, and the ones in row (12), the total contribution for each product. These total contributions, divided by the margin per unit, (4) − (5), yield the final results, quantities to be manufactured (13). The values in row (14) are the corresponding levels of activity for each product.

As an example of application of a different contribution rule, Table 7 shows the results of using the invested capital as the basis for distribution of the burden of facility-sustaining and business-sustaining costs plus profit; that is, Eqs. (7) and (11). The contribution to channel-sustaining costs is computed as in the first solution, (17) = (8).

Note that the target profit in the running example, \$2,131,309, is the 16% of the total invested capital (total in row (19) of Table 7). That is, the target profit may be established in this case in terms of rate of return on invested capital.

4. Production and sales constraints and products with insufficient contribution capability

Section 2 defines (manufacturing) capacity constraints, $Y(x_j)$, $j = 1, 2, \ldots, m$, in terms of production at practical capacity. This variable is used in some of the alternatives for common costs and profit contribution apportionment discussed therein. It is also suggested that the model user may deem appropriate to weight said variable with a coefficient, $0 < \lambda_j \leqslant 1$, $j = 1, 2, \ldots, m$, to take sales constraints into account. In both cases, production and sales limitations are considered only for the purpose of distributing among products the burden of common costs and the target profit. In the running section, these two alternative measures of product contribution capability will be considered in a different way. Namely, what will be discussed is how to factor them into the model, so that it yields feasible solutions.

To address the problem described, plus the one posed by products whose margins are zero or negative, a contribution rule shall include three clauses, in addition to those discussed in Section 2 (Fig. 1), as shown in Fig. 2.

Regarding clause iv, if the business is willing to maintain, for strategic or marketing-related reasons, products with a non-positive margin, the rule would simply state the need to input target output quantities for them. That is

$$\forall x_j | p(x_j) \leqslant v(x_j)$$
$$T(x_j), 1 \leqslant j \leqslant m | 0 \leqslant T(x_j) \leqslant Y(x_j).$$

With reference to clause v, the rule shall state which constraints, on production or on sales, will apply. Consistently, the solutions of the model must verify

$$Q(x_j) \leqslant \lambda(x_j) Y(x_j)$$
$$0 < \lambda(x_j) \leqslant 1, \quad j = 1, 2, \ldots, m.$$

It would not be inconsistent to apply sales constraints here, while production constraints are used to determine products' contributions to common costs and profit.

Finally, the rule must establish (clause vi) how to deal with the contribution deficits arising in products with insufficient contribution capability. For instance, if the decision to drop or maintain a product is not decentralized, it may seem appropriate to establish that contribution deficits be covered by all the products with spare contribution capability, and in the proportion of their contributions to business-sustaining costs and profit. In this case, the algorithm for computation of the model's results would be as shown in Fig. 3, which also takes into account the contents of clauses iv and v.

iv. How will be dealt with in the model those products with a non-positive margin.

v. Whether it will be the sales or the production constraints that will be used to guarantee the feasibility of the solutions to the model.

vi. If a product falls short of covering its specific costs (both fixed and variable) plus its share of common costs and target profit, due to production or sales constraints, how will the resulting contribution deficit be distributed among the other products.

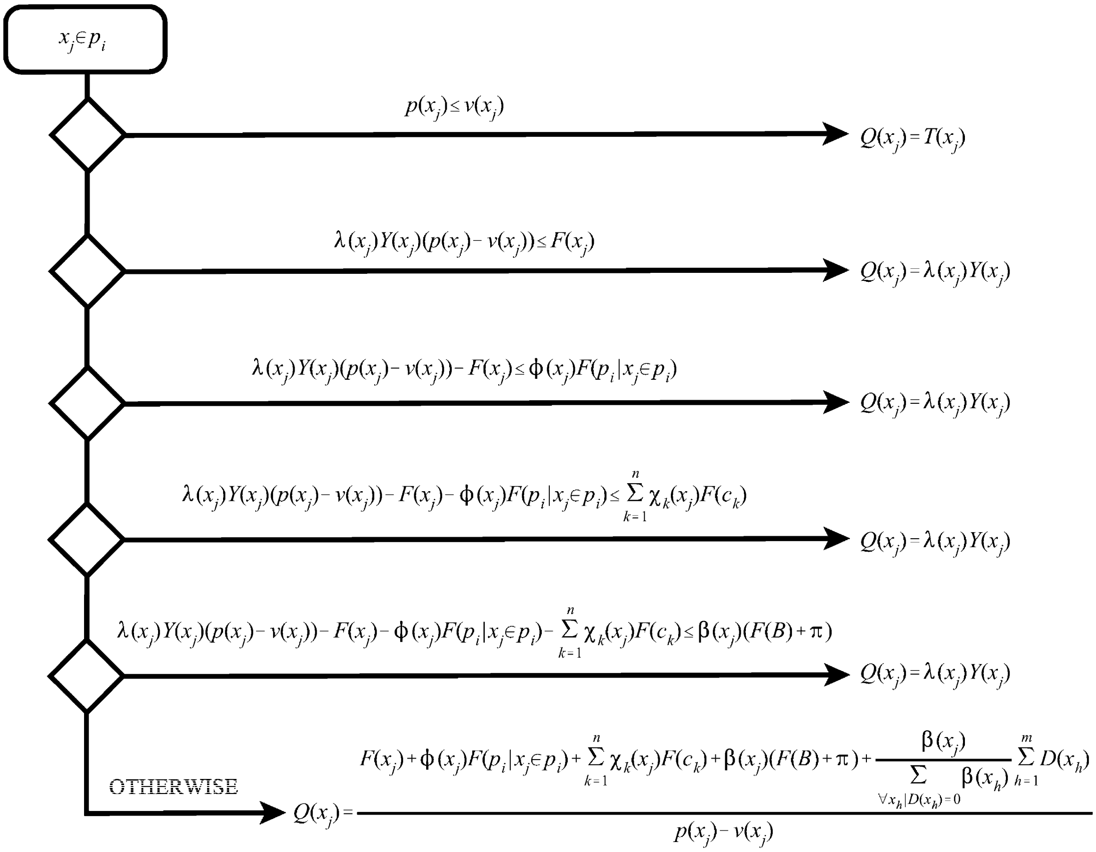
Fig. 2. Contribution rule (production and sales constraints and products with insufficient contribution capability).

5. Manufacturing-related non-flexible limited resources shared by different products

In previous sections, the possible existence of non-flexible resources, associated with batch- and unit-related activities, and applicable to the manufacturing of different products, has not been taken into account for simplicity. The running section addresses that possibility. Consistently with what is argued in Section 1, the straightforward application of linear programming to solve the problem is discarded (on the grounds that it would imply that production is supply-driven). Two alternative solutions are proposed:

i. To apportion the shared resources to the sharing products in proportion to the standard product mix.
ii. To apply linear programming to obtain a synthetic mix yielding a combined margin at capacity that is a weighted average of the results of two mixes of the sharing products: the optimal mix, and the worse possible one when the shared resources are used up.

Let us elaborate on the second alternative. Assume that s products, manufactured at the same facility, share resources: r_h, $h = 1, 2, \ldots, t$, being $F(r_h)$ the fixed costs of resource r_h, $M(r_h)$ the quantity



$$D(x_j) = \begin{cases} F(x_j) + \phi(x_j)F(p_i|x_j \in p_i) + \sum_{k=1}^{n} \chi_k(x_j)F(c_k) + \beta(x_j)(F(B) + \pi) - T(x_j)(p(x_j) - v(x_j)) & , \quad p(x_j) \leq v(x_j) \\ \max\left[F(x_j) + \phi(x_j)F(p_i|x_j \in p_i) + \sum_{k=1}^{n} \chi_k(x_j)F(c_k) + \beta(x_j)(F(B) + \pi) - \lambda(x_j)\, Y(x_j)(p(x_j) - v(x_j)), 0\right], & p(x_j) > v(x_j) \end{cases}$$

$$j = 1, 2, \ldots, m$$

Fig. 3.

available, and $q(r_h, x_j)$, $j = 1, 2, \ldots, s$, the consumption of the resource per unit of each product. The production constraints would now be as follows:

$$Q(x_j) \leqslant Y(x_j), \quad j = 1, 2, \ldots, s,$$

$$\sum_{j=1}^{s} q(r_h, x_j)Q(x_j) \leqslant M(r_h), \quad h = 1, 2, \ldots, t.$$

That is, the production limit imposed individually on each product by the non-shared resources, plus a collective limitation imposed by the availability of the shared resources.

The following algorithm would give the best possible mix:

$$V_1 = \max \sum_{j=1}^{s} Q(x_j)(p(x_j) - v(x_j)),$$

$$Y(x_j) \geqslant Q(x_j) \geqslant 0, \quad j = 1, 2, \ldots, s,$$

$$\sum_{j=1}^{s} q(r_h, x_j)Q(x_j) \leqslant M(r_h), \quad h = 1, 2, \ldots, t.$$

And the worst possible mix, subject to the full use of the shared resources may be obtained solving

$$V_2 = \min \sum_{j=1}^{s} Q(x_j)(p(x_j) - v(x_j)),$$

$$Y(x_j) \geqslant Q(x_j) \geqslant 0, \quad j = 1, 2, \ldots, s,$$

$$\sum_{j=1}^{s} q(r_h, x_j) Q(x_j) = M(r_h), \quad h = 1, 2, \ldots, t.$$

Note that if output quantities have to be integers, then the condition of full use of the shared resources may make the previous algorithm unsolvable. In that case, the following algorithm should be used to compute the worse mix (for a certain minimum use of the shared resources):

$$V_2 = \min \sum_{j=1}^{s} Q(x_j)(p(x_j) - v(x_j)),$$

$$Y(x_j) \geqslant Q(x_j) \geqslant 0 \text{ and } Q(x_j) \in \text{II}, \quad j = 1, 2, \ldots, s,$$

$$\mu M(r_h) \leqslant \sum_{j=1}^{s} q(r_h, x_j) Q(x_j) \leqslant M(r_h), \quad h = 1, 2, \ldots, t,$$

where $0 < \mu < 1$.

Finally, the following algorithm yields the synthetic mix:

$$\sum_{j=1}^{s} Q(x_j)(p(x_j) - v(x_j)) = \alpha V_1 + (1 - \alpha) V_2,$$

$$Y(x_j) \geqslant Q(x_j) \geqslant 0, \quad j = 1, 2, \ldots, s,$$

$$\sum_{j=1}^{s} q(r_h, x_j) Q(x_j) \leqslant M(r_h), \quad h = 1, 2, \ldots t,$$

$$0 \leqslant \alpha \leqslant 1.$$

Where the weights, α and $1 - \alpha$, are the probabilities assigned by the model user to the two extreme mixes as if only one or the other could take place. The following numeric example illustrates the procedure just described.

Table 8 shows the data for two products that share two resources, whose costs and available quantities are disclosed in Table 9.

Table 10 shows the two extreme mixes, the contribution margin resulting from each of them, and the weights assigned by the user of the model.

Finally, Table 11 gives the synthetic mix, the corresponding resources consumption, and the resulting contribution margin.

Therefore, when it is the case, it would be necessary to add another clause to the contribution rule, as indicated in Fig. 4.

6. Summary and directions for further research

This paper has developed a cost-volume-profit model with the following features:

i. it works in a multiproduct situation, yielding unique non-optimal solutions;
ii. it works in the absence of production constraints associated with limited resources shared by the products in the mix, typical of the linear programming-based approach to multiproduct CVPA;

Table 9

Description	Resource 1	Resource 2
Fixed costs	\$2,380,000	\$2,950,000
Units available	80,000	65,000

Table 8

Description	Product 1	Product 2	Product 3
Product fixed costs	\$1,750,000	\$1,200,000	\$1,140,000
Capacity (units) not considering shared resources	50,000	60,000	30,000
Price p.u.	\$140	\$135	\$146
Variable cost p.u.	\$25	\$35	\$28
Margin p.u.	\$115	\$100	\$118
Consumption p.u. of output of shared resource 1	1.1	1.7	1.5
Consumption p.u. of output of shared resource 2	0.95	1.2	0.75

Table 10

Description	Output of product			Consumption of resource		Margin	Weights
	1	2	3	1	2		
Best possible mix	50,000	0	16,666	79,999.00	59,999.50	$7,716,588	60%
Worse possible mix for a minimum use of 95% of shared resources	30,425	27,372	0	79,999.90	61,750.15	$6,236,075	40%

Table 11

Description	Output of product			Consumption of resource		Margin
	1	2	3	1	2	
Synthetic mix	43,527	12,077	7,721	79,992.10	61,633.80	$7,124,382.80

vii. If the manufacturing of one or more products involves the use of common non-flexible limited resources, for modeling purposes the production at capacity of said products will be the one for which:

a) Each product would comsume a quantity of each common resource not bigger than the result of applying the weight of the product in the standard product mix to the quantity available of the resource; or

b) Each product would consume a quantity of each common resource not bigger than the one corresponding to a synthetic mix of the optimal mix, and the worse possible one when all the shared resources are completely consumed.

Fig. 4. Contribution rule (manufacturing-related non-flexible limited resources shared by different products).

iii. it is scalable, meaning that it allows for modeling at enterprise level;
iv. the required input data should be available if an ABC system, designed to keep track of variable and fixed costs, is in place; and
v. it requires the model user to formulate a contribution rule that should be consistent with the characteristics of the business and its operating environment, and that should reflect the model user's judgement regarding the extent to which the different products must contribute to recover the different costs and attain a certain profit.

To conclude, it is acknowledged the need for further research, specially but not exclusively, regarding the effect of using different alternatives for the apportionment of the burden of common costs and the contribution to the target profit. In addition, the model would be enriched if more of such alternatives, other than those described in the previous sections, are devised, and if extensions of the model to industries where CVPA cannot be conducted in terms of output volume are explored.

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